State-Dependent Macroeconomic Policy Effects: A Varying-Coefficient VAR

Adrian Ochs, University of Cambridge
Christian Rörig, University of Cambridge

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Motivation

Research Question

How to estimate state-dependent policy effects in a VAR using a data driven approach?

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How to estimate state-dependent policy effects in a VAR using a data driven approach?

- This is not about identification.
- Assume we perfectly observe a shock series, e.g. government spending shock.

Motivation

Hypothesis:

- Effects of macroeconomic policies are likely heterogeneous, varying across time and economic circumstances.

→ Source of model uncertainty and misspecification bias.

Existing approaches:

- Mostly estimate constant effects of macroeconomic policies.
- State-dependence introduced in a linear fashion using pre-specified dummies.
- Time-varying approaches leave it up for speculation what drives policy effects in specific periods.

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Idea:

- Treat policy effects as a function of economic environment and estimate effects semi-parametrically.

Methodology

Varying coefficient model (VCM)

Often, we estimate:

$$y_t = \beta \varepsilon_t + X_t' \gamma + u_t, \qquad \text{where} \quad \mathbb{E}[u_t | \varepsilon_t, X_t] = 0$$

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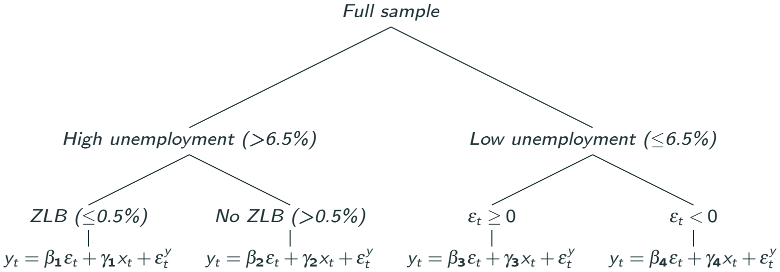
Instead, treat β as a function of the states of the economy:

$$y_t = \beta \varepsilon_t + X_t' \gamma + u_t, \qquad \text{where} \quad \mathbb{E}[u_t | \varepsilon_t, X_t, \Omega_t] = 0$$
$$\beta = f(\Omega_t),$$
$$\gamma = g(\Omega_t),$$

- y_t: macroeconomic outcome variable
- ε_t: exogenous policy shock
- x_t: vector of controls
- β_t: policy coefficient
- γ_t: control coefficient
- Ω_t: economic states/moderators
- u_t: error term, iid.

Macroeconomic state-dependencies as trees

$$\Omega = \{\text{Unemployment}, \text{T-Bill}, \text{Shock Series}\}$$



- Trees offer a natural representation for macroeconomic states and their potential interactions (Goulet Coulombe, 2020).
- They can capture state-dependence and asymmetries.

Tree-based VCM

Tree-based VCM:

$$y_t = \sum_{m=1}^{M} \beta_m I_{(\Omega_t \in C_m)} \varepsilon_t + \boldsymbol{x}_t' \gamma_t + u_t,$$

- $\{C_m\}_{m=1}^{M}$: a partition of the moderator space
- M: number of partitions

Estimation:

$$\left(\hat{C}_m, \hat{\beta}_m\right) = \underset{(C_m, \beta_m)}{\operatorname{argmin}} \sum_{t=1}^{T} \left(y_t - \sum_{m=1}^{M} \beta_m I_{(\Omega_t \in C_m)} \varepsilon_t - \boldsymbol{x}_t' \gamma_t \right)^2$$

- $\{C_m\}_{m=1}^{M}$, M, and β_m are unknown and require simultaneous estimation.
- The estimation of β_m is nested in that of the partitions.
- Within each partition, β_m is simply the least squares estimator on the corresponding sub-sample.

Forest-based VCM

Problems with single trees:

- Tend to overfit
- High variance
- Depend on hyperparameters

Solution: Random forests (Breiman, 2001)

- Average many trees using bootstrapping
- Split on random sample of m (out of p) moderators

$$\beta_{RF}(\Omega_t) = \frac{1}{K} \sum_{k=1}^{K} \sum_{m_k=1}^{M_k} \beta_{m_k} I_{(\Omega_t \in C_{m_k})}.$$

with K trees.

VC-VAR

Consider a state-dependent VAR in structural form:

$$A(\Omega_t)Y_t = BY_{t-1} + \varepsilon_t$$

$$\begin{bmatrix} a_{11} & 0 \\ a_{21}(\Omega_t) & a_{22} \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^y \end{bmatrix}$$

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Reduced form:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21}(\Omega_t) & f_{22}(\Omega_t) \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t^x \\ u_t^y \end{bmatrix}$$

where $A^{-1}(\Omega_t) \equiv Q(\Omega_t)$, $F(\Omega_t) = Q(\Omega_t)B$ and $u_t = Q(\Omega_t)\varepsilon_t$.

$\rightarrow$ Estimate reduced form using forest-based VCM to obtain $F(\Omega_t)$.

VC-VAR

Re-arranging the structural model yields

$$x_t = \frac{b_{11}}{a_{11}} x_{t-1} + \frac{b_{12}}{a_{11}} y_{t-1} + \frac{1}{a_{11}} \varepsilon_t^x$$
$$y_t = -\frac{a_{21}(\Omega_t)}{a_{22}} x_t + \frac{b_{21}}{a_{22}} x_{t-1} + \frac{b_{22}}{a_{22}} y_{t-1} + \frac{1}{a_{22}} \varepsilon_t^y ,$$

$\rightarrow$ Directly estimate the structural equation using a forest-based VCM to obtain relative impact,

$$y_t = \beta_1(\Omega_t) x_t + \beta_2(\Omega_t) x_{t-1} + \beta_3(\Omega_t) y_{t-1} + u_t^y .$$

High-dimensional IRFs: Graphing

- Problem: We have many moderators
- Solution:
  - a: Specific values
  - b: Partial dependency plots
  - Choose a vector of specific moderator values, $\omega^* \in \Omega$.
  - Obtain reduced form matrix, F, as $\hat{F}^h(\omega^*)$.
  - And impact matrix, Q, as $\hat{Q}(\omega^*)\varepsilon$.
  - We assume that economy stays in the particular state. There is no regime-switching.

Then the IRF can be computed as:

$$\widehat{IRF}_{t+h}(\omega^*) = \hat{F}^h(\omega^*)\hat{Q}(\omega^*)\varepsilon$$

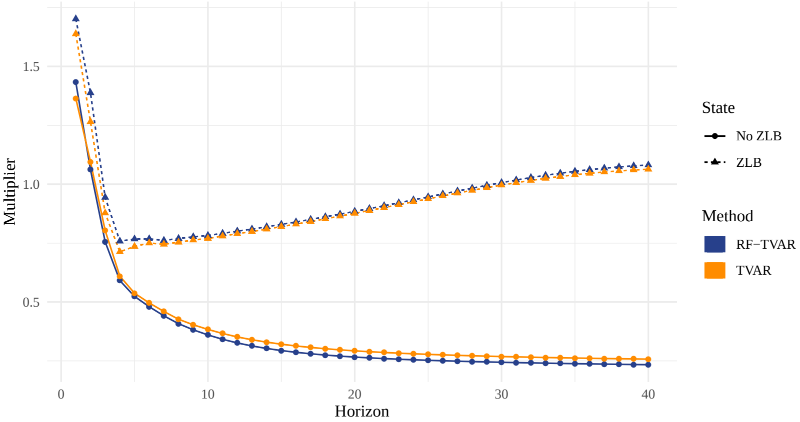
Empirical application

Replicating RZ (2018)

Recap on Ramey and Zubairy (2018):

- Question: Do government spending shocks have different effects in a low vs. high ($> 6.5\%$) unemployment regime, and the zero lower bound?
- Method: LP-IV and TVAR with dummies and military news shocks.
- Findings: Larger multipliers when the economy is close to the ZLB or when the unemployment rate is high.

Sanity check: Using dummy as modifier



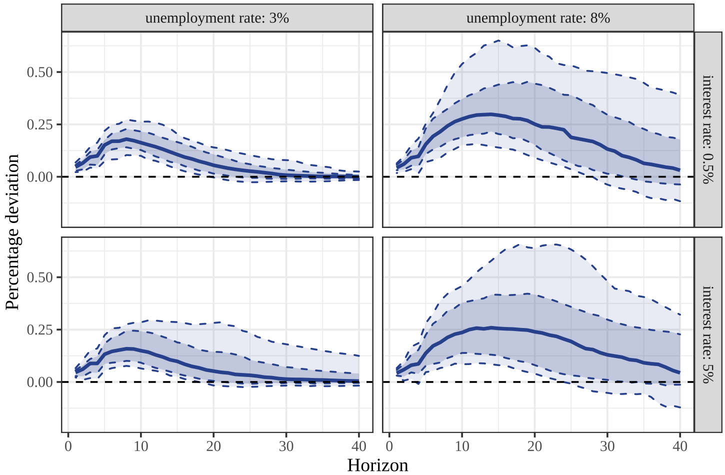
When feeding in the pre-defined dummy, the semi-parametric estimator finds the same result.

Fully flexible estimation

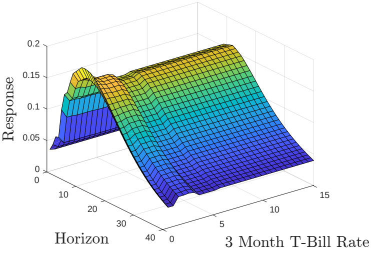
- No pre-defined dummies: $\Omega = \{\text{T-bill rate, Unemployment rate}\}$
- Estimate the following system fully flexible

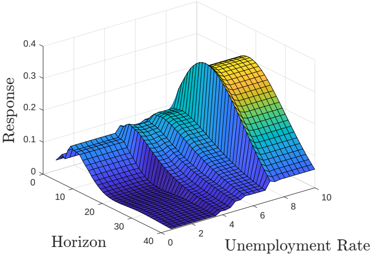
$$\boldsymbol{A}(\Omega) \begin{bmatrix} news_t \\ g_t \\ y_t \end{bmatrix} = \boldsymbol{B}(\Omega) \begin{bmatrix} \mathbb{L}(news_t) \\ \mathbb{L}(g_t) \\ \mathbb{L}(y_t) \end{bmatrix} + \boldsymbol{\varepsilon_t},$$

Sliced IRFs: Effect of Gov. Spending on Output

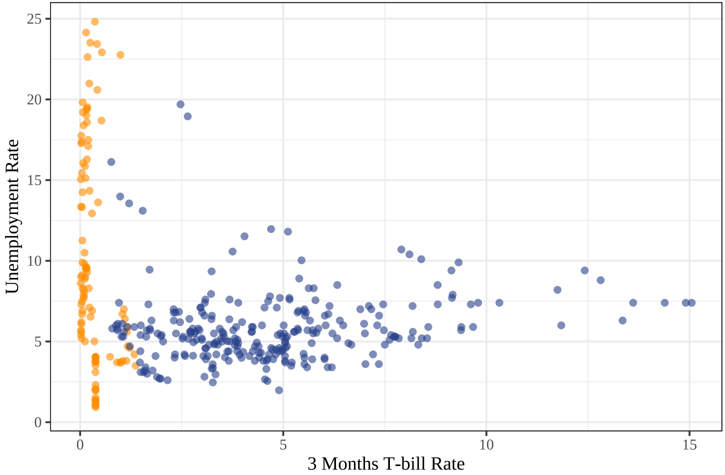


Partial dependency IRFs





Correlation between ZLB and unemployment



Conclusion

- Our approach can detect state/time-dependence without the need to define any priors.
- Hence, it reduces model uncertainty and can also be used to inform the researcher on the parametric model specification for more efficient estimation.
- It offers a more granular perspective on the often ignored high-dimensionality of macroeconomic policy effects.
- The varying coefficient setup offers a great interface to use the power of ML tools in parametric frameworks to keep it interpretable and efficient.

Appendix

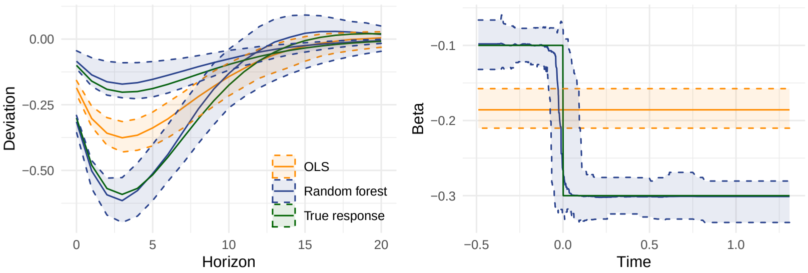
Simulation studies

Endogenous state-dependence

The piece-wise definition of β_t allows introducing all kinds of non-linearities and asymmetries in the policy response for the forest-based estimator to uncover.

$$
\begin{aligned}
i_t &= 0.8 i_{t-1} + 0.1 y_{t-1} + \varepsilon_t^i, \quad \varepsilon_t^i \sim N(0, 0.5^2) \\
y_t &= 0.8 y_{t-1} + \beta_t i_t + \varepsilon_t^y, \quad \varepsilon_t^y \sim N(0, 0.25^2) \\
\beta_t &= \begin{cases} -0.3 & y_{t-1} < 0 \\ -0.1 & y_{t-1} \geq 0 \end{cases}
\end{aligned}
$$

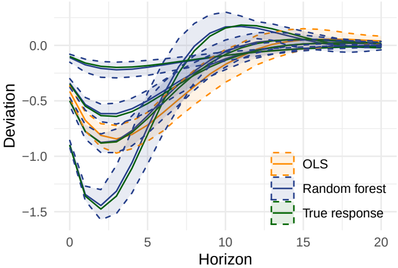
Endogenous state-dependence

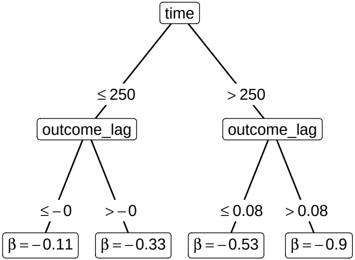


Multiple states

$$
\begin{aligned}
i_t &= 0.8 i_{t-1} + 0.1 y_{t-1} + \varepsilon^i_t, \quad \varepsilon^i_t \sim N(0, 0.5^2) \\
y_t &= 0.8 y_{t-1} + \beta_t i_t + \varepsilon^y_t, \quad \varepsilon^y_t \sim N(0, 0.25^2) \\
\beta_t &= \begin{cases} -0.1 & y_{t-1} < 0, t < T/2 \\ -0.35 & y_{t-1} \geq 0, t < T/2 \\ -0.5 & y_{t-1} < 0, t \geq T/2 \\ -0.9 & y_{t-1} \geq 0, t \geq T/2 \end{cases}
\end{aligned}
$$

Multiple states

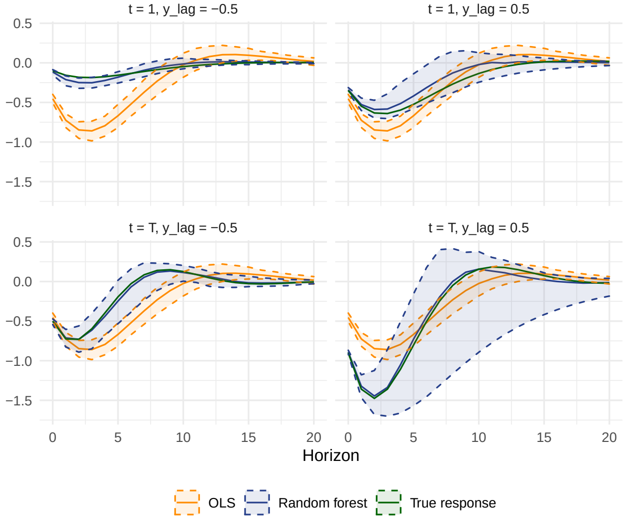




Multiple state-dependent coefficients

$$
\begin{aligned}
i_t &= 0.8 i_{t-1} + \gamma_t y_{t-1} + \varepsilon_t^i, \quad \varepsilon_t^i \sim N(0, 0.5^2) \\
y_t &= 0.8 y_{t-1} + \beta_t i_t + \varepsilon_t^y, \quad \varepsilon_t^y \sim N(0, 0.25^2) \\
\beta_t &= \begin{cases} -0.1 & y_{t-1} < 0, t < T/2 \\ -0.35 & y_{t-1} \geq 0, t < T/2 \\ -0.5 & y_{t-1} < 0, t \geq T/2 \\ -0.9 & y_{t-1} \geq 0, t \geq T/2 \end{cases}, \quad \gamma_t = \begin{cases} 0.3 & y_{t-1} < 0 \\ 0.1 & y_{t-1} \geq 0 \end{cases}
\end{aligned}
$$

Multiple state-dependent coefficients



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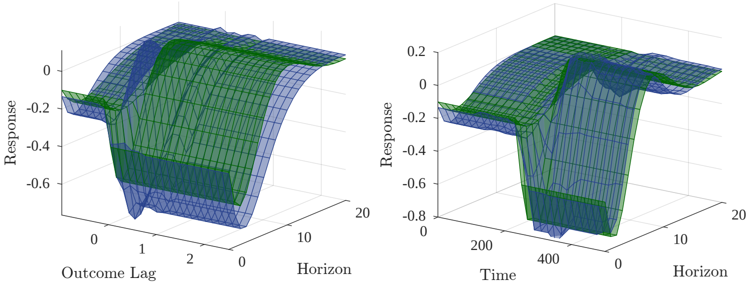

Computational Details

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Algorithm 1 Estimation of varying coefficient model using a random forest following Buergin and Ritschard (2017)

Parameters: T number of trees in forest, e.g., T = 100
N_0 minimum node size, e.g., $N_0 = 30$
D_{min} minimum $-2\cdot$ log-likelihood reduction, e.g., $D_{\text{min}} = 2$
P_{max} maximum levels of pruned tree, e.g., $P_{\text{max}} = 3$

function Random Forest-VCM(S, Ω)
  $H \leftarrow \emptyset$ ▷ Initialize Forest
  for trees in $t = 1$ to T **do**
    $S_t \leftarrow$ A bootstrap sample from the dataset S
    $h_t \leftarrow$ Randomized Tree-VCM(S_t, Ω)
    $H \leftarrow H \cup h_t$
  end for
  return H
end function ▷ Coefficient predictions by averaging over all trees in H

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