

BANK OF ENGLAND

Staff Working Paper No. 907 Banks, shadow banks, and business cycles Yvan Becard and David Gauthier

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Abstract

Credit spreads on household and business loans move in lockstep and spike in every recession. We propose a theory as to why banks tighten their lending standards following a drop in market sentiment. The key feature is a procyclical shadow banking sector that shifts risk from traditional banks to investors through securitisation. We fit the model to euro-area data and find that market sentiment shocks are the main driver of business and financial cycles over the past two decades.

Key words: Credit spreads, shadow banks, business cycles, financial shocks.

JEL classification: E32, E44, G21, G23.

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1 Introduction

What drives business-cycle fluctuations? The view that financial factors are a major cause has become widespread, supported by ever-growing evidence. The literature highlights two main channels through which financial stress impacts the real economy. One involves the role of house prices and credit on *household* spending (Mian and Sufi 2011, 2014, Ramcharan, Verani, and Heuvel 2016, Jensen and Johannesen 2017), the other concerns the disruption of credit on *firm* investment and hiring (Chodorow-Reich 2014, Giroud and Mueller 2017, Huber 2018).

On the theoretical front, researchers have modeled the two channels extensively. Financial frictions in macroeconomic models now range from consumers borrowing to purchase houses and goods (Guerrieri and Lorenzoni 2017, Justiniano, Primiceri, and Tambalotti 2018, Kaplan, Mitman, and Violante 2020) to firms using debt to finance investment projects (Jermann and Quadrini 2012, Arellano, Bai, and Kehoe 2018, Kiyotaki and Moore 2019) to banks collecting deposits to fund the corporate sector (Brunnermeier and Sannikov 2014, Boissay, Collard, and Smets 2016, Gertler, Kiyotaki, and Prestipino 2020). While the relative importance of the two channels is debated (Gertler and Gilchrist 2019, Kehoe et al. 2020), this large body of research has greatly improved our understanding of the interplay between financial crises and recessions.

Yet for all the recent progress, existing theories fail to explain one empirical regularity: credit spreads on household and business loans move hand in hand. This fact, shown in Figure 1 for the euro area, suggests the two channels are always operating together.^{1,2} When banks tighten credit, they tighten for households and firms at once. Figure 1 also displays an (inverted) index of consumer confidence, a leading indicator believed to be a central force behind aggregate spending. Consumer pessimism spikes at the eve of each recession, a few months before spreads themselves shoot up. Thus, it appears that as confidence among consumers and investors sinks, financial markets respond in a way that banks end up restricting credit across the board. Interestingly,

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¹The correlation is 0.96 in the euro area over the 2003-2020 period. Figure 6 in the Online Appendix shows that this fact holds across individual countries of the euro area as well as in the United States.

²To be sure, a few papers do model the two channels together. But they either abstract from spreads (Iacoviello 2005, Lombardo and McAdam 2012) or resort to correlated household- and firm-specific shocks in order to match the joint movements of spreads in the data (Gerali et al. 2010, Ferrante 2019).



Figure 1: Consumer Pessimism and Credit Spreads in the Euro Area

bank deposits tend to increase in bad times, as insured depositors fly to safety, implying that this traditional source of funding does not pose a threat to banks. By contrast, it is well documented that nonbank financial institutions, also known as shadow banks, suffer runs in periods of stress.³ These institutions handle vast amounts of debt and serve as an additional source of funding for commercial banks. But they typically exhibit a volatile and procyclical behavior.

In this paper, we propose a theory as to why banks adjust their lending standards simultaneously on all their borrowers. The key element is a financial sector in which traditional banks interact with shadow banks. Traditional banks (henceforth banks) transform deposits from investors into loans for the real sector of the economy. Shadow banks do not supply loans directly but absorb a large chunk of risk which the banks are unwilling to bear. This is based on the real-world observation that shadow banks provide mainstream banks with an array of services including securitization, insurance, and liquidation of non-performing loans. Such services represent a transfer of risk from original lenders to outside investors and the price at which they are traded depends on how easily shadow banks raise funds on the financial markets.

We first lay out our theory in a simple two-period banking model (Section 2). Banks hold a portfolio of risky assets but are not allowed to default on their deposit liabilities. To mitigate this credit risk, they may either require a sufficiently large amount of physical asset to be posted as collateral or they may shift part of their portfolio to shadow banks.

Notes: Consumer pessimism corresponds to the consumer confidence indicator, inverted and rescaled. Credit spreads are the difference between interest rates on bank loans (to households and firms) and the short-term euro interbank rate. Shaded bars indicate CEPR-dated recessions.

³Shadow banks include money market funds, private equity funds, hedge funds, insurance companies, securities lenders, and structured investment vehicles. Gorton and Metrick (2012), Pozsar et al. (2013) and Covitz, Liang, and Suarez (2013), among others, describe how investors ran on these institutions during the last financial crisis.

Shadow banks pool together many loans, thereby eliminating idiosyncratic risk, and transform them into tradable asset-backed securities, which they sell to investors. We show that securitization enables banks to expand their balance sheet: for a given amount of collateral in the economy, lending is higher.

What happens when confidence among investors dips, say when they realize that securities are riskier than deposits? We refer to this situation as a *market sentiment shock*. Shadow banks suddenly have trouble flogging their securities and must cut back on activity. As the volume of securitization falls, banks compensate by tightening standards, increasing collateral requirements and interest rates. Thus, our theory provides a simple mechanism that links market sentiment to bank credit spreads via the nonbank financial sector.

The next step of our analysis consists in measuring the macroeconomic effects of this propagation mechanism. We embed the framework into a rich dynamic stochastic general equilibrium model designed to fit the European data (Section 3). Borrowers include a subset of households, which employ loans to purchase housing, and entrepreneurs, which employ loans to purchase capital. A fraction of these agents defaults in equilibrium, pushing banks to charge a spread over the risk-free rate. We use macroeconomic and financial data for the euro area from 1999Q1 to 2019Q4 along with standard Bayesian techniques to estimate the parameters of our model.

Our main finding is that a market sentiment shock triggers dynamics that mirror actual business cycles (Section 4). Higher spreads force households and firms to take on fewer loans. House prices and household net worth drop, causing a fall in consumption. Capital prices and firm net worth drop, causing a fall in investment and employment. A recession ensues. The presence of the two channels—on households and firms—is key to replicating the joint behavior of the data. We estimate that the market sentiment shock is responsible for 49 to 55 percent of the variance in output, consumption, investment, and hours worked over the past two decades. The shock also drives most of the movements in the two credit spreads, the nominal interest rate, and is a big force behind credit quantities and house prices. As far as we know, we are the first to assign such a large role in European business and financial cycles to a single disturbance.

To build trust in this story, we perform two external validation exercises (Section 5). First, we compare the time series of the market sentiment shock coming from our estimated model to a measure of systemic financial stress in Europe, which we do not use in the estimation. The two series correlate well, spiking right before the two recessions of the sample. Second, following a recent paper by Angeletos, Collard, and Dellas (2020), we estimate a structural vector autoregression (VAR) where we identify a shock as one that maximizes the volatility in output at business-cycle frequency. We repeat

the procedure by targeting the financial stress index. We show that these two impulses produce virtually identical responses for all endogenous variables, and hence represent two facets of the same "main business-cycle shock". What is more, the dynamics closely resemble those of our structural model when hit by a market sentiment shock. These independent experiments reinforce the credibility of sentiment shocks, or changes in aggregate confidence, as major drivers of business and financial cycles.

Our analysis contributes to a vast literature which estimates structural models to understand economic fluctuations.⁴ As noted above, no study explains the joint dynamics of household and firm credit spreads and quantities with a unique force. One exception is Becard and Gauthier (2020), where we find that a shock to the ability of banks to redeploy collateral ties the spreads together and helps account for the comovements between consumption and investment seen in US data. That shock, however, is exogenous, and should be viewed as a reduced form for deeper events occurring in the financial system. This paper goes a step further and microfounds these very events by modeling the market for securitized debt—hence making the story endogenous. In our model the financial sector becomes a conduit through which waves of optimism or pessimism propagate and amplify.

This article also relates to two active lines of research. The first line emphasizes the role of confidence and expectations on business cycles (Lorenzoni 2009, Angeletos and La'O 2013, Beaudry and Portier 2014, Benhabib, Wang, and Wen 2015, Angeletos, Collard, and Dellas 2018). We do not attempt to model the cause of extrinsic shocks (eg coordination failure, imperfect information, departure from rationality) but rather take these shocks for granted and study their consequences. Our results reveal that sentiment swings can account for the cyclical behavior of a broad range of macroeconomic and financial aggregates. The other line of research focuses on shadow banking. Shadow banks exist because i) they provide liquidity and funding to financial intermediaries (Pozsar et al. 2013, Gennaioli, Shleifer, and Vishny 2013, Moreira and Savov 2017); ii) they benefit from light regulation (Gorton and Metrick 2010, Acharya, Schnabl, and Suarez 2013, Plantin 2015, Begenau and Landvoigt 2020). Our model belongs in the first category, but shares the common insight in this literature that shadow banks make the financial system and the whole economy more vulnerable to reversals in confidence.

2 A Simple Model of Securitization

The economy lasts for two dates, t = 1, 2, and is populated by three types of agents, banks, shadow banks, and investors. We describe them in turn.

⁴Prominent examples include Smets and Wouters (2003, 2007), Justiniano, Primiceri, and Tambalotti (2010), Iacoviello and Neri (2010), Jermann and Quadrini (2012), Schmitt-Grohé and Uribe (2012), Liu, Wang, and Zha (2013), Christiano, Motto, and Rostagno (2014), Ajello (2016), and Bloom et al. (2018).

2.1 Banks

A measure one of banks collect deposits D from investors in period t = 1. Banks use the deposits to extend many loans B to the real sector of the economy, which includes households and firms. Loans are backed by collateral K corresponding to real estate or physical capital. The market price of collateral is Q. We assume K is increasing in B, but concave; in particular, $K(B) = B^{\gamma}$, with $\gamma \in [0, 1]$. This captures the idea that as banks lend more, they are entitled to more collateral, but there is a finite amount of good collateral in the economy, hence the decreasing returns.

Deposits are totally safe and pay the risk-free rate *R* in period t = 2. By contrast, loans are risky. In t = 2, they repay R^b with probability 1-p and default with probability *p*. In case of default, banks seize the collateral whose value depends on a bank-level idiosyncratic shock ω . This shock is undiversifiable by the bank and transforms *QK* units of collateral into ωQK effective units. Let ω_{\min} be the minimum possible value of ω , known by the bank in t = 1.

No Securitization.—Consider first the case where banks do not securitize. Lending to many borrowers turns the ex ante individual default risk into a known fraction of defaulting borrowers. That is, a bank's expected profit in period t = 2 is

$$E[\Pi^{ns}] = (1-p)R^bB + pE[\omega]QK(B) - RD.$$

To ensure it always repays its depositors, the bank must not take on more deposit liabilities than its assets are worth under the worst-case scenario

$$(1-p)R^{b}B + p\omega_{\min}QK(B) \ge RD.$$

Setting this solvability constraint to equality and using the balance sheet B = D, we obtain the maximum amount of lending under the no-securitization scenario

$$B^{ns} = \left(\frac{R - (1 - p)R^b}{p\omega_{\min}Q}\right)^{\frac{1}{\gamma - 1}}$$

To see whether the solvability constraint binds, maximize profit subject only to B = Dand obtain optimal lending, $B^{ns\star} = \left(\frac{R-(1-p)R^b}{p\gamma E[\omega]Q}\right)^{\frac{1}{\gamma-1}}$. Assuming $\omega_{\min} < \gamma E[\omega]$, we have $B^{ns\star} > B^{ns}$. Therefore, the bank always prefers lending more than it is allowed to, ie the constraint binds.

With Securitization.—Suppose that in period t = 1 each bank has the possibility to sell part of its loan book to a shadow bank at price $\tilde{Q} > \omega_{\min}Q$. This enables the bank

to offload its idiosyncratic risk and guarantee instead a safe revenue stream. Under this arrangement, the bank's expected profit in t = 2 writes

$$E[\Pi^{s}] = (1 - p)R^{b}B + p\tilde{Q}K(B) - RD.$$
 (1)

Using B = D and maximizing profit, we obtain

$$B^{s} = \left(\frac{R - (1 - p)R^{b}}{p\gamma\tilde{Q}}\right)^{\frac{1}{\gamma - 1}}.$$
(2)

The condition $R - (1 - p)R^b > 0$ ensures that profit and lending are bounded. Next, as long as $E[\omega] \leq \tilde{Q}/Q$, we find that for any $\{\omega_{\min}, \gamma\} \in [0, 1)$, expected profit is larger when banks securitize.⁵ Therefore, it is always in the interest of the bank to engage in securitization. This leads us to our first result.

Proposition 1. For a given interest rate R^b , and provided that $\gamma \tilde{Q} > \omega_{min}Q$, the balance sheet of banks is larger with securitization, $B^s > B^{ns}$.

2.2 Shadow Banks

A measure one of shadow banks buy many claims K from banks at price \tilde{Q} and bundle them into asset-backed securities S. The securities are issued to investors in period t = 1and promise to repay R^s in period t = 2. Pooling together a large number of claims eliminates the bank-level risk. Let ω_{mean} be the average value of ω over all defaulting borrowers. The balance sheet of the shadow bank in period t = 1 is

$$\omega_{\text{mean}}\tilde{Q}K(B) = S$$

In period t = 2, the market value of these claims is $\omega_{\text{mean}}QK(B)$. Profit is then $\omega_{\text{mean}}QK(B) - R^s \omega_{\text{mean}}\tilde{Q}K(B)$. Profit maximization yields

$$R^s = \frac{Q}{\tilde{Q}}.$$
(3)

⁵Expected profit when behaving optimally under the two scenarios writes

$$\begin{split} E[\Pi^{ns}] &= \left[(1-p)R^b - R \right] \left(\frac{R - (1-p)R^b}{p\omega_{\min}Q} \right)^{\frac{1}{\gamma-1}} + pE[\omega]Q \left(\frac{R - (1-p)R^b}{p\omega_{\min}Q} \right)^{\frac{\gamma}{\gamma-1}},\\ E[\Pi^s] &= \left[(1-p)R^b - R \right] \left(\frac{R - (1-p)R^b}{p\gamma\tilde{Q}} \right)^{\frac{1}{\gamma-1}} + p\tilde{Q} \left(\frac{R - (1-p)R^b}{p\gamma\tilde{Q}} \right)^{\frac{\gamma}{\gamma-1}}. \end{split}$$

2.3 Investors

A measure one of investors receive a perishable endowment W in period 1 and wish to consume in both periods. Their utility is

$$E[U(C_1) + U(C_2)],$$

where C is consumption and U is an increasing and concave function. The budget constraints are

$$C_1 = W - D - S;$$
 $E[C_2] = RD + E[R^s]S.$

Optimization yields $E[M]R = E[MR^s]$, where $M \equiv U'(C_2)/U'(C_1)$ is the stochastic discount factor. Define the covariance term $\nu \equiv -Cov[M, R^s]/E[M]$ and rewrite the first-order condition as

$$E[R^s] = R + \nu, \tag{4}$$

We assume that v is exogenous and refer to it as the *market sentiment shock*.

2.4 Equilibrium

So far we have said nothing about the economy's ultimate borrowers, except that they default with probability p. In the next section's extended model, the actions taken by these agents will result in endogenous default probability, loan interest rate, and collateral value. For now, we simply assume that the demand for loans B is a decreasing function of the loan interest rate R^b .

Combining the first-order conditions from banks (2), shadow banks (3), and investors (4), we get

$$R + \nu = \frac{p\gamma Q B^{\gamma - 1}}{R - (1 - p)R^b}.$$
(5)

All else being equal, an increase in ν either leads to fall in *B* or an increase in R^b . Equation (5) brings us to our second, and key, result.

Proposition 2. In equilibrium, a deterioration in market sentiment, that is a rise in v, reduces the amount of securities S and loans B and increases the loan interest rate R^b .

The intuition is as follows. Lower sentiment makes investors more risk averse, prompting them to demand a higher return on risky securities. Shadow banks cut back on securitization, driving down the price of securitized assets \tilde{Q} . To compensate this drop in income, banks must either reduce their balance sheet by lending less or increase their spread $R^b - R$, which in turn reduces the demand for loans.

3 Extension and Estimation

Having illustrated how investor sentiment affects credit spreads in a parsimonious environment, we turn to a quantitative dynamic general equilibrium model designed to fit the European data. There are two types of households, patient and impatient. Patient households are the economy's ultimate savers and correspond to the investors of the previous section. Impatient households are net debtors: they obtain loans from banks to purchase housing and consume. In the business sector, entrepreneurs also obtain loans from banks to purchase physical capital. These two types of borrowers are subject to agency problems, similar to Bernanke, Gertler, and Gilchrist (1999, hereafter BGG), and a fraction of them defaults each period. Financial institutions act as the middlemen, as before.

We embed this framework into a standard model of business cycles such as the one estimated by Smets and Wouters (2007). Although some features like price indexation and adjustment costs have been criticized for lacking supporting micro evidence, they help the model match the persistence of macro aggregates.

3.1 The Extended Model

We describe the nonstandard elements and relegate the well-known parts to the Online Appendix.

Households.—Denote patient and impatient households by superscripts p and i, respectively. Each household contains a large number of workers indexed by $k \in [0, 1]$ and enjoys lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^{o,t} \left\{ \zeta_{c,t} \ln(C_t^o - b_c^o C_{t-1}^o) + \ln H_t^o - \psi_l \int_0^1 \frac{l_{k,t}^{o,1+\sigma_l}}{1+\sigma_l} dk \right\}, \quad o \in \{p,i\},$$

where C_t^o denotes consumption, H_t^o denotes housing services, $l_{k,t}^o$ is specialized labor, and $\zeta_{c,t}$ a is preference shock. The parameter b_c^o determines habit, ψ_l is a weight coefficient, and σ_l is the inverse elasticity of labor supply. To ensure that impatient households are net borrowers, we impose $\beta^i < \beta^p$. The patient household's budget constraint is

$$(1+\tau^{c})P_{t}C_{t}^{p} + Q_{t}^{h}\bar{H}_{t}^{p} + P_{t}D_{t} + P_{t}S_{t} \leq (1-\tau^{l})\int_{0}^{1}W_{k,t}^{p}l_{k,t}^{p}dk + R_{t-1}P_{t-1}D_{t-1} + v_{t-1}^{-1}R_{t-1}^{s}P_{t-1}S_{t-1} + Q_{t}^{h}\bar{H}_{t-1}^{p} + \Delta_{t}^{p} + T_{t}^{p},$$

where P_t is the price of final goods, \bar{H}_t^p is a housing good that provides H_t^p units of housing services, Q_t^h is the price of housing, D_t is deposits, S_t is securities, $W_{k,t}^p$ is the

nominal wage of worker k, R_t is the nominal risk-free rate, R_t^s is the interest rate on securities, Δ_t^p bundles dividends from firms and shadow banks, T_t^p is a transfer from the government, and τ^c and τ^l are consumption and labor tax rates. As in the previous section, v_t is a *market sentiment shock*, the decisive exogenous variable in our analysis.

Impatient Households.—The budget constraint of impatient workers is

$$(1+\tau^{c})P_{t}C_{t}^{i}+P_{t}r_{t}^{h}H_{t}^{i} \leq (1-\tau^{l})\int_{0}^{1}W_{k,t}^{i}l_{k,t}^{i}dk+\Delta_{t}^{i}+T_{t}^{i},$$

where r_t^h is the rental rate of housing and dividends Δ_t^i are described below.

Besides workers, the impatient households comprises a large number of homeowners.⁶ At time t, a homeowner combines her net worth N_t^i and a loan B_t^i from a bank to acquire housing from housing good producers. She receives a standard debt contract and promises to repay $R_{t+1}^i B_t^i$ in the next period. At the start of period t + 1, each homeowner is hit by an idiosyncratic shock ω^i drawn from a unit-mean lognormal distribution with cumulative distribution F^i . At this point, net worth is given by

$$N_{t+1}^{i} = R_{t+1}^{h} \omega^{i} Q_{t}^{h} \bar{H}_{t}^{i} - R_{t+1}^{i} B_{t}^{i},$$

where $R_{t+1}^h \equiv Q_{t+1}^h / Q_t^h$ is the return on housing. The budget constraint is

$$Q_{t+1}^{h}\bar{H}_{t+1}^{i} + \Delta_{t+1}^{i} = N_{t+1}^{i} + P_{t+1}r_{t+1}^{h}\bar{H}_{t+1}^{i} + B_{t+1}^{i}$$

The goal of the homeowner is to maximize the dividend Δ_{t+1}^i she pays to her family subject to the budget constraint and a bank participation constraint, given below. A default threshold $\bar{\omega}_{t+1}^i$ separates homeowners who are able to pay off their debt from those who are not

$$R_{t+1}^{h}\bar{\omega}_{t+1}^{i}Q_{t}^{h}\bar{H}_{t}^{i} = R_{t+1}^{i}B_{t}^{i}.$$

Finally, we assume housing adjustment costs similar to investment adjustment costs.

Entrepreneurs.—There is a large number of entrepreneurs (superscript *e*). These are analogous to homeowners so far as their relationship with the bank is concerned. Each combines her net worth N_t^e and a loan B_t^e to acquire capital from capital producers

$$Q_t^k \bar{K}_t = N_t^e + B_t^e,$$

⁶We split the impatient household into workers and homeowners to ensure that the problem of the borrowing agent—the homeowner—is linear in net worth, which facilitates aggregation (Ferrante, 2019).

where Q_t^k is the market price of capital. After being hit by an idiosyncratic shock ω^e drawn from distribution F^e , the entrepreneur earns revenues by renting out capital services $\omega^e K_t$ to productive firms and selling depreciated capital back to capital producers after production. Following Christiano, Motto, and Rostagno (2014), we let σ_t^e denote the standard deviation of log ω^e and refer to it as the firm risk shock. The return per unit of capital is

$$R_t^k = \left[(1 - \tau^k) [u_t r_t^k - a(u_t)] \Upsilon^{-t} P_t + (1 - \delta) Q_t^k + \tau^k \delta Q_{t-1}^k \right] / Q_{t-1}^k,$$

where u_t is capital utilization, a is a utilization cost, and τ^k is tax on capital. The object $\Upsilon > 1$ accounts for investment-specific technical change, ie final goods convert into $\Upsilon^t \mu_{\Upsilon,t}$ investment goods, where $\mu_{\Upsilon,t}$ is a shock.

Similarly to homeowners, entrepreneurs default if the cost of servicing debt exceeds the value of collateral, $R_{t+1}^k \bar{\omega}_{t+1}^e Q_t^k \bar{K}_t = R_{t+1}^e B_t^e$. The goal of an entrepreneur in period *t* is to maximize expected net worth

$$E_t \int_{\bar{\omega}_{t+1}^e}^{\infty} [R_{t+1}^k \omega^e Q_t^k \bar{K}_t - R_{t+1}^e B_t^e] dF^e(\omega^e),$$

subject to a participation constraint set by the bank.

Banks.—A representative, competitive bank transforms deposits from patient households into mortgage loans B_t^i to impatient households and business loans B_t^e to entrepreneurs. Loans are backed by collateral—housing $Q_t^h \bar{H}_t^i$ for mortgages and capital $Q_t^k \bar{K}_t$ for business loans. As in the two-period model, the bank insures itself against any loss by transferring the risky part of its loan portfolio to a shadow bank. Thus, Equation (1) of the previous section, together with the zero-profit condition, duplicates into a pair of participation constraints which the bank imposes on its borrowers, one for each type

$$[1 - F^{i}(\bar{\omega}_{t+1}^{i})]R_{t+1}^{i}B_{t}^{i} + F^{i}(\bar{\omega}_{t+1}^{i})\tilde{Q}_{t+1}^{h}\bar{H}_{t}^{i} \ge R_{t}B_{t}^{i}, \tag{6}$$

$$[1 - F^{e}(\bar{\omega}_{t+1}^{e})]R^{e}_{t+1}B^{e}_{t} + F^{e}(\bar{\omega}_{t+1}^{e})\tilde{Q}^{k}_{t+1}\bar{K}_{t} \ge R_{t}B^{e}_{t}.$$
(7)

The left-hand side corresponds to the bank's revenues. Its second term makes clear that part of these revenues hinge on the prices of asset-backed securities, \tilde{Q}_{t+1}^h and \tilde{Q}_{t+1}^k , which themselves depend on the demand for ABS coming from shadow banks. This connects the traditional and shadow banking sectors and is what differentiates the constraints from their original formulation in BGG.

Shadow Banks.—The representative shadow bank employs funding S_t from patient households to acquire mortgage and business loan portfolios from mainstream banks.

Its budget constraint in period t is

$$S_t = F^i(\bar{\omega}_t^i)\tilde{Q}_t^h\bar{H}_{t-1}^i + F^e(\bar{\omega}_t^e)\tilde{Q}_t^k\bar{K}_{t-1}.$$

As the claim owner, the shadow bank is entitled to the return on the underlying assets, namely housing $G^i(\bar{\omega}^i) \equiv \int_0^{\bar{\omega}^i_t} \omega^i dF^i(\omega^i) R^h_t Q^h_{t-1} \bar{H}^i_{t-1}$, and capital $G^e(\bar{\omega}^e) \equiv \int_0^{\bar{\omega}^e_t} \omega^e dF^e(\omega^i) R^k_t Q^k_{t-1} \bar{K}_{t-1}$. We assume the shadow bank enjoys some degree of market power in the two markets for asset-backed securities, in the form of markups, μ^h and μ^k . These parameters replace the monitoring cost μ in the standard financial accelerator mechanism of BGG. In Online Appendix Section A, we discuss the differences between our modified framework and the original costly state verification model.

Shadow bank profit is transferred to patient households as dividends. Thus, the shadow bank maximizes after-dividend profit

$$(1-\mu^{h})G^{i}(\bar{\omega}_{t}^{i})R_{t}^{h}Q_{t-1}^{h}\bar{H}_{t-1}^{i}+(1-\mu^{k})G^{e}(\bar{\omega}_{t}^{e})R_{t}^{k}Q_{t-1}^{k}\bar{K}_{t-1}-R_{t}^{s}S_{t},$$

subject to its budget constraint. The first-order condition equalizes the marginal benefit of the two assets

$$(1-\mu^{h})\frac{R_{t}^{h}Q_{t-1}^{h}}{\tilde{Q}_{t}^{h}}\frac{G^{i}(\bar{\omega}_{t}^{i})}{F^{i}(\bar{\omega}_{t}^{i})} = (1-\mu^{k})\frac{R_{t}^{k}Q_{t-1}^{k}}{\tilde{Q}_{t}^{k}}\frac{G^{e}(\bar{\omega}_{t}^{e})}{F^{e}(\bar{\omega}_{t}^{e})}.$$

This equation binds together the prices of mortgage-backed and capital-backed securities. The securitized debt market thus acts as a centralizing force that synchronizes asset prices.

Other Agents.—The rest of the model is presented and derived in Online Appendix Section B, where we also list all the equilibrium conditions.

Shocks.—We consider 12 shocks: the market sentiment shock, permanent and transitory technology, permanent and transitory investment-specific technology, preference, housing, markup, firm equity, firm risk, government spending, and monetary policy. All have the same structure and follow the process $\ln(x_t/x) = \rho_x \ln(x_{t-1}/x) + \varepsilon_t^x$, with $\varepsilon^x \sim N(0, \sigma_x)$.



Figure 2: The Role of the Market Sentiment Shock

3.2 Estimation

We estimate the model on quarterly aggregate data for 12 countries of the euro area.⁷ The sample period is 1999Q1–2019Q4. We use 11 series: GDP, consumption, investment, work hours, inflation, the nominal interest rate, household credit, business credit, household spread, business spread, and house prices.⁸ A number of parameters are calibrated based on our data set and other targets. We estimate the remaining parameters with Bayesian techniques. The Online Appendix provides a detailed description of the data and its treatment (Section C), the calibration and estimation of parameters, and measures of model fit (Section D).

4 The Market Sentiment Shock

This section presents our main findings. We examine various indicators which reveal the leading role of sentiment shocks on European business cycles. We then describe the mechanisms at play in our model that explain this result.

4.1 The Predominant Market Sentiment Shock

Consider Figure 2. To isolate the effects of the market sentiment shock on the economy, we feed it to our model while shutting off other shocks. We then compare the outcome

⁷The countries are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain. As of 2019Q4, they account for 97.6 percent of the entire 19-country euro area's GDP.

⁸Credit spreads are only available from 2003Q1 onward.

(solid line) to the actual data (dashed line). The key takeaway is that the model hit with just one impulse does a great job at matching the historical behavior of the main macroeconomic and financial variables. This is all the more striking given that we estimate twelve shocks in total, and only when all of them are active does the model replicate the data exactly. The match is far from perfect, though, and leaves ample room for the other forces. For example, the model overshoots the boom in economic activity preceding the 2008 financial crisis. We find that contractionary monetary policy shocks were pulling the economy down during that period. The model also misses part of the difference. Notwithstanding, we believe it is remarkable that a single disturbance generates factual dynamics for as many real and financial variables, spanning different sectors, including prices and quantities. The fit for the two credit spreads, in particular, is impressive.

Another indicator of the importance of market sentiment shocks appears in Table 1. The table reports the contribution of the different shocks to the variance in our eleven observable variables at business-cycle frequency (6-32 quarters). According to this criterion, the market sentiment shock is the most important force driving the business and financial cycle. It accounts for the bulk of the variance in output (55 percent), consumption (49), investment (49), hours (50), the nominal interest rate (74), the household spread (89), and the firm spread (83). It is also a sizable impulse behind house prices (31), household credit (23), and firm credit (32). These last two variables are steered by disturbances specific to their sector, namely the housing shock for household credit and the entrepreneurial equity shock for business credit. Also, inflation is mostly driven by markup shocks. We will see that this variable barely responds to a market sentiment shock realization. This is consistent with evidence in Angeletos, Collard, and Dellas (2020) on US data, which we repeat in the next section for European data, that prominent business-cycle shocks, whatever their nature, do not appear to be inflationary.

Taken together, our results suggest that shocks to the relative return between risky and safe assets are a major source of economic fluctuations in Europe. To the best of our knowledge, we are the first to attribute such a consistently high share of variation in the main macroeconomic and financial aggregates to a single force.

4.2 Why is the Market Sentiment Shock So Important?

To answer this question we turn to impulse responses. Figure 3 plots the reaction of the model economy to a negative realization of the market sentiment shock. Think of a risk-off moment in the financial system. Investors fly to safety and demand a higher yield on their riskier investments, which happen to be the asset-backed securities issued

	Sentiment v_t	Technology $\varepsilon_t, \mu_{z^*,t}$	Investment $\zeta_{I,t}, \mu_{\Upsilon,t}$	Household $\zeta_{c,t}, \zeta_{h,t}$	Firm $\lambda_{p_t}, \gamma_t^e, \sigma_t^e$	Policy g_t, ε_t^p
Output	55	8	16	4	10	7
Consumption	49	13	4	28	5	1
Investment	49	6	29	1	15	0
Hours worked	50	17	14	4	9	6
Inflation	30	17	1	1	51	1
Nominal rate	74	5	6	2	10	4
Household credit	23	20	0	53	3	0
Firm credit	32	21	8	0	38	0
Household spread	89	5	0	3	2	0
Firm spread	83	0	3	0	13	0
House price	31	45	6	10	6	1

Table 1: Variance Decomposition, 6-32 Quarters

Note: The variance decomposition is computed on bandpass-filtered data generated by the model evaluated at the mode of the posterior distribution.

by shadow banks. These institutions are forced to scale down their balance sheet. The price of securitized loans plunges to clear the market. This translates into losses for banks, which compensate by increasing the interest rate they charge to their borrowers, households and firms alike. The shock then propagates to these sectors of the economy.

On the consumer side, credit drops as the spread widens. Impatient households are forced to cut on consumption and housing purchases. Lower demand for real estate reduces the price of housing. This hurts the net worth of borrowers, who become more prone to default. Banks respond by further increasing lending rates, exacerbating the situation. On the business side, a similar story takes place. Entrepreneurs are forced to cut on capital purchases. Investment drops. Labor becomes superfluous and hours fall. A lower demand for capital reduces its price. This hurts the net worth of entrepreneurs, who are charged higher rates on their debt, and must therefore reduce investment further. Less consumption and investment leads to a fall in output. Inflation also falls, but quantitatively the response is subdued. The central bank responds by lowering the policy rate.

In short, a market sentiment shock in our model induces procyclical consumption, investment, employment, inflation, nominal interest rate, household credit, business credit, asset prices; countercyclical household and business credit spreads. These dynamics are precisely the ones we observe in the European data. That explains why our empirical analysis assigns such a large role to market sentiment shocks in business and financial cycles.



Figure 3: Response to a Negative Market Sentiment Shock

5 External Validation

This article claims that variations in market sentiment are responsible for the lockstep motion of credit spreads and the bulk of macroeconomic fluctuations. Taking this claim seriously requires to have faith in the shock itself as well as in the transmission mechanism of the underlying model. In this section, we offer support for each of these elements based on external data and a different econometric approach.

5.1 The Market Sentiment Shock and Financial Stress

Under a literal interpretation, a market sentiment shock in our stylized model is an exogenous change to the risk premium commanded by asset-backed securities over safe deposits. We prefer a broader interpretation, under which the shock reflects a reversal of aggregate sentiment in the financial sector. To back this view, we compare in Figure 4 the market sentiment shock process coming out of our estimated model to a measure of systemic financial stress in Europe. That measure is the Composite Indicator of Systemic Stress (CISS) constructed by the European Central Bank. It aggregates many indicators from money, bond, equity, and foreign exchange markets, as well bank-specific data on return volatility and credit spreads. We emphasize that this series was not used in the inference about our model's parameters.

The chief observation is that our shock correlates well with the measure of financial stress. The contemporaneous correlation coefficient is 0.67 over the sample period. The market sentiment shock spikes in three occasions, during the 2001 dot-com bubble (which did not lead to a recession in Europe), the 2008 financial crisis, and the 2012 sovereign debt crisis. The same is true for the CISS, with a little lag. Interestingly, the



Figure 4: Financial Stress, Model Versus Data

correlation is at its highest, 0.72, when the shock leads the indicator by one quarter. A test of predictive causality indicates that the market sentiment shock Granger causes the financial stress index at the 0.1-percent confidence level, at up to six lags. We conclude that our theoretical object is a fairly good gauge of financial market sentiment.

5.2 European Business-Cycle Anatomy

In a recent paper, Angeletos, Collard, and Dellas (2020) develop a new strategy to analyze business cycles. They estimate a VAR on a number of US aggregate series where just one shock is identified so as to maximize the volatility of one particular series over a particular frequency band. They repeat the exercise for each variable, hence taking multiple cuts of the data and providing an "anatomy." Their main result is interchangeability: whether one targets GDP, consumption, investment, hours worked, or unemployment, the different shocks produce nearly the same dynamic comovements in all variables of interest. The authors consider these shocks to be the various facets of a single force, which they label the "main business-cycle shock".

Our goal in this subsection is to construct a European main business-cycle shock and compare its dynamic properties to those of our market sentiment shock. We follow the procedure described by Angeletos, Collard, and Dellas (2020). The data is the same as in the estimation of our structural model, except that we add the CISS presented above as an additional observable variable. Also, we enter all variables in levels, that is we don't detrend or demean any series. We estimate a VAR with two lags using Bayesian methods and a Minnesota prior. We then construct a structural shock as a linear combination of the VAR residuals. The identification criterion requires that the shock maximize the

Notes: Financial stress corresponds to the European Central Bank's Composite Indicator of Systemic Stress (CISS). The market sentiment shock is scaled to fit the figure.



Figure 5: Market Sentiment Shock Versus Main Business-Cycle Shock

Notes: Impulse responses to the market sentiment shock are scaled to match those of the VAR. The shaded area corresponds to the 68% highest posterior density interval around the responses to the output-based main business-cycle shock.

contribution to the volatility of a particular variable over a particular frequency band. That variance is computed in the frequency domain, and the chosen band is between 6 and 32 quarters. We target two variables successively: output and the CISS. Figure 5 displays the response of the VAR to these two shocks, along with the response of our structural model to the estimated market sentiment shock.

Two findings emerge from the inspection of Figure 5. First, the two VAR-based shocks are virtually indistinguishable. They both cause procyclical movements in consumption, investment, hours, inflation, nominal rate, and credit quantities, as well as countercyclical movements in spreads. Thus, it appears that in the euro area over the last two decades, the main business-cycle shock, defined as one that maximizes the variation in output, coincides with the "main financial shock", defined as one that maximizes the variation in systemic financial stress.

The second finding is that the market sentiment shock gives rise to almost the same impulse responses as the (two versions of the) main business-cycle shock. All endogenous variables exhibit a nice hump-shaped behavior, peaking after four to six quarters, except for the two credit spreads, which are monotonous. The model's responses are as persistent as those of the VAR. This is thanks to the shock process's relatively low autocorrelation coefficient of 0.939, a value smaller than most estimates found in the literature for financial disturbances.⁹ Thus, the anatomy test vindicates our model's transmission mechanism. In this model, shocks to investor confidence trigger dynamics that resemble those obtained from a much more flexible statistical framework. Given how different the two approaches are, the proximity in their outcome provides further

⁹For example, the risk shock in Christiano, Motto, and Rostagno (2014) has an autocorrelation coefficient of 0.972. Jermann and Quadrini (2012) find 0.969 for their financial shock.

support to the market sentiment shock as a leading impulse to business cycles.

6 Conclusion

Credit spreads on household and business loans move hand in hand. This is important because spreads, a yardstick for lending conditions, are closely in tune with the business cycle. This paper offers a theory that links investor sentiment to credit spreads via the nonbank financial sector. In our model, shadow banks operate alongside commercial banks to securitize risky individual loans and hence produce standardized asset-backed securities. Investors perceive these securities, free of any idiosyncratic risk, to be nearly as safe as traditional bank deposits, and consequently purchase them. That, in turn, allows banks to expand lending by charging lower spreads.

In periods of stress, however, the "nearly" qualification turns out to be crucial and the imperfect substitution between securities and deposits grows apparent. Securities suddenly command a higher premium, enough to curtail the capacity of shadow banks to engage in securitization. This spills over to commercial banks: no longer able to offload part of their portfolio at the same price, they resort to increasing spreads on consumers and businesses alike.

How does that affect the real economy? As spreads shoot up, credit becomes dearer. Indebted households must cut back on goods and housing purchases. Indebted firms must cut back on capital purchases. Employment, consumption and investment fall, causing a recession. Thus, a drop in investor confidence—we call it a market sentiment shock—produces strong and positive comovements among the main macroeconomic variables, credit quantities, and asset prices, as well as countercyclical movements in household and business credit spreads. These implications of the model correspond well to the behavior of actual European business and financial cycles.

We estimate our model using eleven aggregate series for the euro area. We find that the market sentiment shock is the main driver of economic and financial fluctuations since the eurozone exists. It accounts particularly well for the two recessions in 2009 and 2012. In addition, the shock process correlates well with a measure of systemic stress, a piece of information that was not used in the estimation. Finally, following a recent methodological contribution by Angeletos, Collard, and Dellas (2020), we show that the dynamics produced by the market sentiment shock are similar to those implied by a VAR-based main business-cycle shock.

Business cycles in Europe and the United States are highly synchronized. A promising avenue of research would be to model these economies in a two-country setup and study how market sentiment shocks that emanate from one country, say the US, propagate to the other country through their financial linkages.

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Online Appendix to "Banks, Shadow Banks, and Business Cycles"

Yvan Becard and David Gauthier

This appendix is divided into four sections. Section A provides additional results. Section B derives the extended model and lists all equilibrium equations. Section C describes the data. Section D discusses the estimation of the parameters.

A Additional Evidence

A.1 Credit Spreads in Euro Area Countries and the United States

As mentioned in the Introduction, Figure 6 plots the evolution of household and business credit spreads in twelve member states of the euro area as well as in the United States. In every country except Greece, the series are highly correlated. In all instances, spreads spike during each recession of the sample. Thus, coordinated and countercyclical spreads is a stylized fact of business cycles in advanced economies.

A.2 Switching Off the Shadow Banks

The main contribution of this paper is to devise a mechanism that rationalizes correlated credit spreads. We argue that the nonbank financial sector, with its volatile and procyclical funding structure, is a crucial conduit through which investor confidence propagates to the mainstream banking sector, and then, to the rest of the economy.

One way to highlight the role played by our mechanism is to deactivate it. We estimate a version of the model without shadow banks. Traditional banks continue intermediating funds between patient households on the one hand and impatient households and entrepreneurs on the other. But now we make the standard assumption that they are able to perfectly diversify idiosyncratic risk. This renders securitization moot: in case of default, banks simply seize the pledged collateral and sell it back at market price. The financial friction resides in a bankruptcy cost banks must pay to audit their borrowers. Thus, the version without shadow banks boils down to a standard costly state verification model a la BGG, but with two financial accelerator mechanisms—on households and firms—instead of one. Specifically, the banks' participation constraints



Figure 6: Household and Firm Credit Spreads in Euro Area Countries and the United States

Notes: Credit spreads in the euro area are the difference between interest rate on bank loans and the short-term euro interbank rate. Spreads in the United States are with respect to the federal funds rate. Numbers in bracket indicate the correlation between the two series. Shaded bars show recessions.

(Equations (6) and (7) in the main text) recover their usual form

$$\begin{split} & [1 - F^{i}(\bar{\omega}_{t+1}^{i})]R_{t+1}^{i}B_{t}^{i} + (1 - \mu^{h})G^{i}(\bar{\omega}_{t+1}^{i})R_{t+1}^{h}Q_{t}^{h}\bar{H}_{t}^{i} \geq R_{t+1}B_{t}^{i}, \\ & [1 - F^{e}(\bar{\omega}_{t+1}^{e})]R_{t+1}^{e}B_{t}^{e} + (1 - \mu^{k})G^{e}(\bar{\omega}_{t+1}^{e})R_{t+1}^{k}Q_{t}^{k}\bar{K}_{t} \geq R_{t+1}B_{t}^{e}, \end{split}$$

where μ^h and μ^k denote the monitoring costs of impatient households and entrepreneurs, respectively.

We estimate the alternative model on the same data set, using the exact same prior distributions as in the baseline case. Note that the market sentiment shock becomes irrelevant in the absence of its propagation mechanism, so we drop it. Instead, we add a household risk shock σ_t^i , mirroring the firm risk shock σ_t^e . In particular, σ_t^i denotes the standard deviation of $\log \omega^i$ and is meant to reflect idiosyncrasies in local housing markets (local employment, public infrastructure, weather conditions, population dynamics).¹⁰

With the market sentiment shock out of the race, we find that the firm risk shock

¹⁰We first estimated our baseline model with the household risk shock. But its contribution to all variables, including the household spread, was close to nil, so we decided to drop it.



Figure 7: Shock Contribution to Credit Spreads

becomes the main driving force of the European business cycle.¹¹ This echoes the results in Christiano, Motto, and Rostagno (2014) with US data. However, the firm risk shock struggles to match a number of variables related to households, most importantly consumption, credit quantity, and *the credit spread*. As it happens, the household risk shock (partially) fills the gap. Figure 7 summarizes these findings by decomposing the contribution of prominent shocks to the evolution of the household and firm credit spreads. The top row presents the outcome for the baseline model, ie with the market sentiment shock, while the bottom row shows the equivalent for the model without shadow banks.

The two economies are in stark contrast. In our baseline model, the market sentiment shock makes up the lion's share of both credit spreads movements. As explained in the main text, this is because banks rely on shadow banks as part of their risk management strategy. When investor sentiment deteriorates, shadow banks retrench from the securitized loan market, pushing down the price of asset-backed securities, and obliging banks to offset this drop in income by increasing lending rates on household

Notes: Solid lines correspond to actual credit spreads in the euro area from 2003Q1 to 2019Q4. Vertical bars show the contribution of the different shocks to the evolution of spreads. Dashed lines correspond to credit spreads implied by the estimated model between 1999Q1 and 2002Q4, period for which data on spreads is not available.

¹¹The firm risk shock accounts for 31, 9, 42, and 22 percent of the variance in output, consumption, investment, and hours, respectively. Parameter estimates and a complete variance decomposition are available upon request.

	Market sentiment shock v_t	Firm risk shock σ_t^e	Household risk shock σ_t^i	
A. Baseline Model				
Variance household spread	89	5	_	
Variance firm spread	85	0	_	
Covariance H spread–F spread	99	0	_	
B. Model with No Shadow Banks				
Variance household spread	-	0	81	
Variance firm spread	-	92	0	
Covariance H spread–F spread	-	-243	-784	

Table 2: Contribution of Three Shocks, 6-32 Quarters

Note: Variance and covariance contributions are computed on bandpass-filtered data generated by the two models evaluated at the mode of the posterior distribution.

and business borrowers altogether. In the model without shadow banks, the two risk shocks emerge as the central force behind credit spreads. But each risk shock accounts for its 'own' spread: the household risk shock drives the household spread, the firm risk shock drives the firm spread. Neither disturbance has any role whatsoever on the 'other' spread. The upshot is that, in order to match the joint dynamics of credit spreads in the data, the estimation procedure attributes a high degree of correlation of 0.69 between the exogenous processes. This result is at odds with the assumption that shocks are structural and independent from one another.

Table 2 reinforces these points. We report the contribution of sentiment and risk shocks to the variance and covariance in household and firm credit spreads at business-cycle frequency. As the top panel makes clear, the market sentiment shock in the baseline model accounts for virtually all the covariance in spreads. In the model without shadow banks (bottom panel), each risk shock contributes *negatively* to that covariance.

To conclude, absent the transmission channel that shadow banks constitute, the theory is unable to generate a factual response of credit spreads with a single impulse, and therefore falls short of providing a plausible explanation for business cycles. This underscores the contribution of our paper to the debate.

B Derivation of the Extended Model

B.1 Patient Households

Let Λ_t^p be the patient household's marginal utility. The first-order conditions with respect to consumption, housing services, deposits, and asset-backed securities are

$$\begin{split} 0 &= \Lambda_t^p (1 + \tau^c) P_t - \zeta_{c,t} / (C_t^p - b_c^p C_{t-1}^p) + b_c^p \beta^p E_t \zeta_{c,t+1} / (C_{t+1}^p - b_c^p C_t^p), \\ 0 &= 1 / H_t^p - \Lambda_t^p Q_t^h + \beta^p E_t \Lambda_{t+1}^p Q_{t+1}^h, \\ 0 &= \Lambda_t^p P_t - \beta^p P_t E_t \Lambda_{t+1}^p R_t. \\ 0 &= \Lambda_t^p P_t - \beta^p P_t E_t \Lambda_{t+1}^p R_t^s / v_t. \end{split}$$

B.2 Impatient Households

Workers.—Let Λ_t^i be the impatient household's marginal utility. The first-order conditions for consumption and housing services are

$$0 = \Lambda_t^i (1 + \tau^c) P_t - \zeta_{c,t} / (C_t^i - b_c^i C_{t-1}^i) + b_c^i \beta^i E_t \zeta_{c,t+1} / (C_{t+1}^i - b_c^i C_t^i),$$

$$0 = 1/H_t^i - \Lambda_t^i P_t r_t^h.$$

Homeowners.—A homeowner maximizes the present discounted value of dividends

$$V_{t}^{i} = \max_{\bar{H}_{t}^{i}, B_{t}^{i}} \left\{ \Delta_{t}^{i} + \beta^{i} E_{t} \Lambda_{t+1}^{i} / \Lambda_{t}^{i} \max\{0, V_{t+1}^{i}\} \right\},$$

subject to $N_{t}^{i} = R_{t}^{h} \omega^{i} Q_{t-1}^{h} \bar{H}_{t-1}^{i} - R_{t}^{i} B_{t-1}^{i},$
 $Q_{t}^{h} \bar{H}_{t}^{i} + \Delta_{t}^{i} = N_{t}^{i} + P_{t} r_{t}^{h} \bar{H}_{t}^{i} + B_{t}^{i},$

and the bank participation constraint. Substitute the two constraints in the value function and define $\eta_t^i \equiv B_t^i / \bar{H}_t^i$ and $g_t^i \equiv \bar{H}_t^i / \bar{H}_{t-1}^i$

$$V_t^i = \max_{\bar{H}_t^i, B_t^i} \left\{ [R_t^h \omega^i Q_{t-1}^h - R_t^i \eta_{t-1}^i + (P_t r_t^h + \eta_t^i - Q_t^h) g_t^i] \bar{H}_{t-1}^i + \beta^i E_t \Lambda_{t+1}^i / \Lambda_t^i \max\{0, V_{t+1}^i\} \right\}.$$

The value function V_t^i is linearly homogeneous in housing \bar{H}_{t-1}^i . Therefore, all homeowners select the same leverage and default threshold regardless of their housing net worth. We can rewrite the problem in the form of the scaled value function $v_t^i \equiv V_t^i / \bar{H}_{t-1}^i$

$$v_t^i = \max_{g_t^i, \eta_t^i} \left\{ R_t^h \omega^i Q_{t-1}^h - R_t^i \eta_{t-1}^i + (P_t r_t^h + \eta_t^i - Q_t^h) g_t^i + g_t^i \beta^i E_t \Lambda_{t+1}^i / \Lambda_t^i \int_{\bar{\omega}_{t+1}^i}^{\infty} v_{t+1}^i dF^i(\omega^i) \right\}.$$

The first-order conditions with respect to g_t^i and η_t^i are

$$\begin{split} 0 &= P_t r_t^h + \eta_t^i - Q_t^h + \beta^i E_t \Lambda_{t+1}^i / \Lambda_t^i \int_{\bar{\omega}_{t+1}^i}^{\infty} v_{t+1}^i dF^i(\omega_{t+1}^i), \\ 1 &= \beta^i E_t \Lambda_{t+1}^i / \Lambda_t^i [1 - F^i(\bar{\omega}_{t+1}^i)] (R_{t+1}^i + \eta_t^i \partial R_{t+1}^i / \partial \eta_t^i). \end{split}$$

Substitute the optimal condition for g_t^i into the value function and multiply by \bar{H}_{t-1}^i

$$V_t^i = \{R_t^h \omega^i Q_{t-1}^h \bar{H}_{t-1}^i - R_t^i B_{t-1}^i\} = N_t^i.$$

A default threshold $\bar{\omega}_t^i$ is such that $N_t^i < 0$, that is assets are worth less than liabilities

$$R^h_t \bar{\omega}^i_t Q^h_{t-1} \bar{H}^i_{t-1} = R^i_t B^i_{t-1}$$

Now, rewrite the participation constraint

$$R_{t+1}^{i} = \frac{R_{t}\eta_{t}^{i} - F^{i}(\bar{\omega}_{t+1}^{i})\tilde{Q}_{t+1}^{h}}{[1 - F^{i}(\bar{\omega}_{t+1}^{i})]\eta_{t}^{i}}.$$

Compute the partial derivative $\partial R_{t+1}^i / \partial \eta_t^i$ and plug it into the optimal condition for η_t^i

$$1 = \beta^{i} E_{t} \Lambda_{t+1}^{i} / \Lambda_{t}^{i} \left[R_{t} - F^{i\prime}(\bar{\omega}_{t+1}^{i}) R_{t+1}^{i} \tilde{Q}_{t+1}^{h} / (R_{t+1}^{h} Q_{t}^{h}) + \bar{\omega}_{t+1}^{i} F^{i\prime}(\bar{\omega}_{t+1}^{i}) R_{t+1}^{i} \right].$$

Real Estate Broker.—A competitive real estate broker acts as a middleman by purchasing housing goods from housing producers and selling them to the homeowners. In the process of acquiring vast amount of real estate, the broker is subject to housing adjustment costs. These costs are important because they smooth the dynamics of housing and hence of household credit, which is an observable variable, and thus help our model fit the data. The real estate broker maximizes profit

$$E_0 \sum_{t=0}^{\infty} \beta^{i,t} \Lambda_t^i \left\{ Q_t^h \bar{H}_t^i - Q_t^h \bar{H}_t^i \left[1 + S^h(\zeta_{h,t} \bar{H}_t^i / \bar{H}_{t-1}^i) \right] \right\},\,$$

where $S^{h}(t)$ is an increasing convex function and $\zeta_{h,t}$ is a housing shock. The first-order condition is

$$0 = \Lambda_t^i Q_t^h \left[S^h(t) + \zeta_{h,t} \frac{\bar{H}_t^i}{\bar{H}_{t-1}^i} S^{h\prime}(t) \right] + \beta^i E_t \Lambda_{t+1}^i Q_{t+1}^h \zeta_{h,t+1} \left(\frac{\bar{H}_{t+1}^i}{\bar{H}_t^i} \right)^2 S^{h\prime}(t+1) \,.$$

B.3 Entrepreneurs

Define leverage as assets over equity, $L_t^e \equiv Q_t^k \bar{K}_t / N_t^e$, and let $\Gamma^e(\bar{\omega}_{t+1}^e)$ be the expected gross share of entrepreneurial returns going to creditors

$$\Gamma^{e}(\bar{\omega}^{e}_{t+1}) \equiv [1 - F^{e}(\bar{\omega}^{e}_{t+1})]\bar{\omega}^{e}_{t+1} + G^{e}(\bar{\omega}^{e}_{t+1}), \quad G^{e}(\bar{\omega}^{e}_{t+1}) \equiv \int_{0}^{\bar{\omega}^{e}_{t+1}} \omega^{e}_{t+1} dF^{e}(\omega^{e}_{t+1}).$$

Expected pre-dividend net worth is

$$E_t \int_{\bar{\omega}_{t+1}^e}^{\infty} [R_{t+1}^k \omega^e Q_t^k \bar{K}_t - R_{t+1}^e B_t^e] dF^e(\omega^e) = E_t [1 - \Gamma^e(\bar{\omega}_{t+1}^e)] R_{t+1}^k L_t^e N_t^e.$$

Define $\Gamma_2^e(\bar{\omega}_{t+1}^e) \equiv [1 - F^e(\bar{\omega}_{t+1}^e)]\bar{\omega}_{t+1}^e + F^e(\bar{\omega}_{t+1}^e)$ and rewrite the participation constraint using the definitions of leverage and default cutoff

$$\Gamma_2^e(\bar{\omega}_{t+1}^e) + \left(\frac{\tilde{Q}_{t+1}^k}{Q_t^k R_{t+1}^k} - 1\right) F^e(\bar{\omega}_{t+1}^e) = \frac{L_t^e - 1}{L_t^e} \frac{R_t}{R_{t+1}^k}.$$

The problem of an entrepreneur in period t is to choose a pair of leverage and default cutoff $(L_t^e, \bar{\omega}_{t+1}^e)$ to maximize expected net worth in t+1 subject to the bank participation constraint. Since current net worth N_t^e does not appear in the constraint and is present in the objective only as a factor of proportionality, all entrepreneurs select the same $(L_t^e, \bar{\omega}_{t+1}^e)$ regardless of their net worth. Maximization yields

$$0 = E_t \left[1 - \Gamma^e(\bar{\omega}_{t+1}^e) \right] \frac{R_{t+1}^k}{R_t} - E_t \frac{\Gamma^{e'}(\bar{\omega}_{t+1}^e)}{\Gamma_2^{e'}(\bar{\omega}_{t+1}^e) + [\tilde{Q}_{t+1}^k/(Q_t^k R_{t+1}^k) - 1] F^{e'}(\bar{\omega}_{t+1}^e)} \frac{1}{L_t^e}.$$

Utilization Rate.—The entrepreneur also determines the utilization rate of capital u_t . Since the market for capital services is competitive, the user cost function is equal to the return on renting out capital services

$$P_t \Upsilon^{-t} a(u_t) \omega_t^e \bar{K}_{t-1} = P_t \tilde{r}_t^k u_t \omega_t^e \bar{K}_{t-1}.$$

Optimal utilization implies

$$a'(u_t) = \Upsilon^t \tilde{r}_t^k = r^k \exp(\sigma_a[u_t - 1]).$$

B.4 Productive Sector

Final Good Producers.—A representative, competitive final good firm combines intermediate goods $Y_{j,t}$, $j \in [0, 1]$, to produce final output Y_t using the technology

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{1}{\lambda_{p,t}}} dj\right]^{\lambda_{p,t}}$$

where $\lambda_{p,t} \ge 1$ is a markup shock. The firm's budget constraint is $\int_0^1 P_{j,t} Y_{j,t} dj = P_t Y_t$. Optimization leads to the familiar demand function and aggregate price index

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}} Y_t; \qquad P_t = \left[\int_0^1 P_{j,t}^{\frac{1}{1-\lambda_{p,t}}} dj\right]^{1-\lambda_{p,t}}$$

Intermediate Good Producers.—Each intermediate good *j* is produced by a monopolist according to the production function

$$Y_{j,t} = \max\left\{\varepsilon_t (u_t K_{j,t-1})^{\alpha} (z_t l_{j,t})^{1-\alpha} - \theta z_t^* \ ; \ 0\right\}, \quad \alpha \in (0,1),$$

where ε_t is a stationary technology shock and θ is a fixed cost. There are two sources of growth in the model, namely a growth trend in technology z_t and an investmentspecific shock $\mu_{\Upsilon,t}$ that changes the rate at which final goods are converted into $\Upsilon^t \mu_{\Upsilon,t}$ investment goods, with $\Upsilon > 1$. The fixed cost θ is proportional to $z_t^* = z_t \Upsilon^{(\frac{\alpha}{1-\alpha})t}$, which combines the two trends. The intermediate good producer faces standard Calvo frictions. Every period, a fraction $1-\xi_p$ of intermediate firms sets its price $P_{j,t}$ optimally. The remaining fraction follows an indexation rule $P_{j,t} = \pi^{\iota_p} \pi_{t-1}^{1-\iota_p} P_{j,t-1}$, where $\iota_p \in (0, 1)$ and $\pi_t \equiv P_t/P_{t-1}$ is inflation. A variable without the subscript *t* denotes its steady-state value.

Labor input of firm j is a combination of patient and impatient labor

$$l_{j,t} = l_{j,t}^{p,\kappa} l_{j,t}^{i,1-\kappa}, \quad \kappa \in (0,1].$$

Parameter κ is decisive: if $\kappa = 1$ we are back to a representative agent model. Profit writes $P_{j,t}Y_{j,t} - P_t \tilde{r}_t^k u_t \bar{K}_{j,t-1} - W_t^p l_{j,t}^p - W_t^i l_{j,t}^i$, where $P_t \tilde{r}_t^k$ denotes the nominal rental rate of capital. Cost minimization implies

$$\begin{split} P_t \tilde{r}_t^k &= M C_t \alpha \varepsilon_t (u_t \bar{K}_{t-1})^{\alpha - 1} (z_t l_t^{p,\kappa} l_t^{i,1-\kappa})^{1-\alpha}, \\ W_t^p l_t^p &= M C_t (1-\alpha) \kappa \varepsilon_t (u_t \bar{K}_{t-1})^{\alpha} (z_t l_t^{p,\kappa} l_t^{i,1-\kappa})^{1-\alpha}, \\ W_t^i l_t^i &= M C_t (1-\alpha) (1-\kappa) \varepsilon_t (u_t \bar{K}_{t-1})^{\alpha} (z_t l_t^{p,\kappa} l_t^{i,1-\kappa})^{1-\alpha}, \end{split}$$

where MC_t is the multiplier on the production function, ie the marginal cost. We have dropped the *j* subscript because all firms choose the same proportion of inputs and hence share a common marginal cost.

Turning to prices, the intermediate goods producer chooses an optimal price $P_{j,t}$ to maximize the present value of future profits

$$\max_{P_{j,t}} E_t \sum_{s=0}^{\infty} \xi_p^s \beta^{p,s} \Lambda_{t+s}^p Y_{j,t+s} (P_{j,t} \tilde{\Pi}_{t,t+s} - MC_{t+s}),$$

subject to the demand function. Here, $\tilde{\Pi}_{t,t+s} \equiv \prod_{k=1}^{s} \tilde{\pi}_{t+k}$ and $\tilde{\pi}_{t} = \pi^{\iota_{p}} \pi_{t-1}^{1-\iota_{p}}$. Let $\Pi_{t,t+s} \equiv \prod_{k=1}^{s} \pi_{t+k}$. Note the firm uses the discount factor of the patient household, its owner. The first-order condition is

$$0 = E_t \sum_{s=0}^{\infty} \xi_p^s \beta^{p,s} \Lambda_{t+s}^p Y_{t+s} \left(\frac{\tilde{P}_t \tilde{\Pi}_{t,t+s}}{P_t \Pi_{t,t+s}} \right)^{\frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}}} \frac{1}{1-\lambda_{p,t+s}} \left[\tilde{\Pi}_{t,t+s} - \lambda_{p,t+s} \frac{MC_{t+s}}{\tilde{P}_t} \right].$$

The optimal price $\tilde{P}_t \equiv P_{j,t}$ depends only on aggregate variables and is therefore common to all producers. Rearranging, we obtain $\tilde{P}_t = P_t K_{p,t} / F_{p,t}$ where

$$\begin{split} K_{p,t}^{p} &\equiv P_{t}\Lambda_{t}^{p}Y_{t}\frac{\lambda_{p,t}}{1-\lambda_{p,t}}\frac{MC_{t}}{P_{t}} + \xi_{p}\beta^{p}E_{t}\left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_{p,t+1}}{1-\lambda_{p,t+1}}}K_{p,t+1}^{p},\\ F_{p,t}^{p} &\equiv P_{t}\Lambda_{t}^{p}Y_{t}\frac{1}{1-\lambda_{p,t}} + \xi_{p}\beta^{p}E_{t}\left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_{p,t+1}}}F_{p,t+1}^{p}. \end{split}$$

Labor Contractors.—A representative, competitive labor contractor aggregates specialized labor services $l_{k,t}^o$, where $k \in [0,1]$ and $o \in \{p,i\}$, into homogeneous labor l_t^o using the technology

$$l_t^o = \left[\int_0^1 l_{k,t}^{o,\frac{1}{\lambda_w}} dk\right]^{\lambda_w}, \quad o \in \{p,i\}, \, \lambda_w \ge 1.$$

Its budget constraint is $\int_0^1 W_{k,t}^o l_{k,t}^o dk = W_t^o l_t^o$, $o \in \{p, i\}$. Optimization leads to the demand function for intermediate labor and the aggregate wage index

$$l_{k,t}^{o} = \left(\frac{W_{k,t}^{o}}{W_{t}^{o}}\right)^{\frac{A_{W}}{1-\lambda_{W}}} l_{t}^{o}; \qquad W_{t}^{o} = \left[\int_{0}^{1} W_{k,t}^{o,\frac{1}{1-\lambda_{W}}} dk\right]^{1-\lambda_{W}}; \qquad o \in \{p,i\}.$$

Monopoly Unions.—Each worker of type k is represented by a monopoly union that sets its nominal wage rate $W_{k,t}^o$, where $o \in \{p, i\}$. All unions are subject to Calvo frictions in a similar fashion to intermediate firms. A fraction $1 - \xi_w$ of monopoly

unions chooses its wage optimally. The remaining fraction follows an indexation rule $W_{k,t}^o = \mu_{z^*} \pi^{\iota_w} \pi_{t-1}^{1-\iota_w} W_{k,t-1}^o$, where $o \in \{p, i\}$, $\iota_w \in (0, 1)$, $\mu_{z^*} \equiv z^*/z_{-1}^*$ is the steady-state growth rate of the economy, and $\mu_{z^*,t}$ is a shock. An optimizing union maximizes

$$E_{t} \sum_{s=0}^{\infty} \xi_{w}^{s} \beta^{o,s} \left[-\psi_{l} \int_{0}^{1} \frac{l_{k,t+s}^{o,1+\sigma_{l}}}{1+\sigma_{l}} dk + \Lambda_{t+s}^{o} (1-\tau^{l}) W_{k,t}^{o} \tilde{\Pi}_{t,t+s}^{w} l_{k,t+s}^{o} \right], \quad o \in \{p,i\},$$

subject to the demand function. Here, $\tilde{\Pi}_{t,t+s}^{w} \equiv \prod_{k=1}^{s} \mu_{z^*} \tilde{\pi}_{w,t+k}$ and $\tilde{\pi}_{w,t} = \pi^{\iota_w} \pi_{t-1}^{1-\iota_w}$. Let $\Pi_{t,t+s}^{w} \equiv \prod_{k=1}^{s} \pi_{w,t+k}$. The optimal wage condition is

$$0 = E_t \sum_{s=0}^{\infty} \xi_w^s \beta^{o,s} l_{t+s}^o \left(\frac{\tilde{W}_t^o \tilde{\Pi}_{t,t+s}^w}{W_t^o \Pi_{t,t+s}^w} \right)^{\frac{\lambda_w}{1-\lambda_w}} \left[\Lambda_{t+s}^o (1-\tau^l) \tilde{\Pi}_{t,t+s}^w - \frac{\psi_l \lambda_w}{\tilde{W}_t^o} \left(\frac{\tilde{W}_t^o \tilde{\Pi}_{t,t+s}^w}{W_t^o} \right)^{\frac{\lambda_w \sigma_l}{1-\lambda_w}} l_{t+s}^{o,\sigma_l} \right].$$

The optimal wage $\tilde{W}_t^o \equiv W_{k,t}^o$ is common to all worker unions. That is, there is one optimal wage \tilde{W}_t^p for patient workers and another \tilde{W}_t^i for impatient workers. Rearranging, we obtain $\frac{\tilde{W}_t^o}{W_t^o} = \left[\frac{\psi_l}{W_t^o/P_t} \frac{K_{w,t}^o}{F_{w,t}^o}\right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_l)}}$, where $o \in \{p, i\}$ and

$$\begin{split} K^{o}_{w,t} &\equiv l_{t}^{o,1+\sigma_{l}} + \xi_{w}\beta^{o}E_{t}(\tilde{\pi}_{w,t+1}\pi_{w,t+1}^{-1}\mu_{z^{*}})^{\frac{\lambda_{w}}{1-\lambda_{w}}(1+\sigma_{l})}K^{o}_{w,t+1}, \\ F^{o}_{W,t} &\equiv (1-\tau^{l})\lambda_{w}^{-1}l_{t}^{o}P_{t}\Lambda_{t}^{o} + \xi_{w}\beta^{o}E_{t}\left(\tilde{\pi}_{w,t+1}\mu_{z^{*}}\right)^{\frac{1}{1-\lambda_{w}}}\pi_{w,t+1}^{\frac{\lambda_{w}}{\lambda_{w}-1}}\pi_{t+1}^{-1}F^{o}_{W,t+1}. \end{split}$$

Capital Producers.—A representative, competitive capital producer builds raw capital according to a standard technology

$$\bar{K}_t = (1 - \delta)\bar{K}_{t-1} + \left[1 - S^k(\zeta_{i,t}I_t/I_{t-1})\right]I_t, \quad \delta \in (0, 1),$$

where I_t is investment, $S^k(t)$ is an increasing function defined below, and $\zeta_{i,t}$ is a shock to the marginal efficiency of investment. Optimal investment implies

$$0 = \Lambda_t^p Q_t^k \left[1 - S^k(t) - \zeta_{i,t} \frac{I_t}{I_{t-1}} S^{k\prime}(t) \right] - \frac{\Lambda_t^p P_t}{\Upsilon^t \mu_{\Upsilon,t}} + \beta^p E_t \Lambda_{t+1}^p Q_{t+1}^k \zeta_{i,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S^{k\prime}(t+1) .$$

Housing Producers.—Housing is in fixed supply. The total housing stock is

$$\bar{H} = \bar{H}_t^p + \bar{H}_t^i.$$

B.5 Government

The monetary authority follows a standard Taylor rule

$$R_t - R = \rho_p(R_{t-1} - R) + (1 - \rho_p) \left[\alpha_\pi(E_t \pi_{t+1} - \pi) + \alpha_{\Delta y}(g_{y,t} - \mu_{z^*}) \right] + \varepsilon_t^p, \rho_p \in (0, 1),$$

where $\alpha_{\pi}, \alpha_{\Delta y} > 0$ are weights, $g_{y,t}$ is quarterly GDP growth in deviation from steady state, and ε_t^p is a monetary policy shock. The fiscal authority collects taxes to finance public expenditures G_t and transfer lump-sum amounts T_t to households

$$G_t + T_t = \tau^k ([u_t r_t^k - a(u_t)] \Upsilon^{-t} P_t - \delta Q_{t-1}^k) K_{t-1} + \tau^l (W_t^i l_t^i + W_t^p l_t^p) + \tau^c P_t C_t.$$

Government spending is given by $G_t = z_t^* g_t$, where g_t is an exogenous-spending shock.¹² Transfers are distributed to both types of households according to their respective share in total labor income, $T_t = \kappa T_t^p + (1 - \kappa)T_t^i$.

B.6 Aggregation and Market Clearing

Production.—Clearing in the goods market imposes

$$Y_t = G_t + C_t + \Upsilon^{-t} \mu_{\Upsilon,t}^{-1} I_t + a(u_t) \Upsilon^{-t} \bar{K}_{t-1},$$

where $C_t \equiv C_t^p + C_t^i$ is total consumption. We define GDP as $Y_t^{\text{gdp}} \equiv G_t + C_t + \Upsilon^{-t} \mu_{\Upsilon,t}^{-1} I_t$.

Impatient Households.—As explained above, all homeowners choose the same leverage and default threshold. Perfect insurance within the household ensures they begin the next period with the same level of net worth, which in aggregate is given by

$$N_{t}^{i} = [1 - \Gamma^{i}(\bar{\omega}_{t}^{i})]R_{t}^{h}Q_{t-1}^{h}\bar{H}_{t-1}^{i}, \quad \Gamma^{i}(\bar{\omega}_{t}^{i}) \equiv [1 - F^{i}(\bar{\omega}_{t}^{i})]\bar{\omega}_{t}^{i} + G^{i}(\bar{\omega}_{t}^{i}).$$

Entrepreneurs.—The quantity of physical capital produced by capital producers must equal the quantity purchased by entrepreneurs, $\bar{K}_t = \int_0^1 \bar{K}_{j,t} dj$. The aggregate supply of capital services provided by entrepreneurs must equal the demand from intermediate firms, $K_t = \int_0^1 K_{j,t} dj$. Since ω^e has unit mean, that supply is

$$K_t = \int_0^1 \int_0^\infty u_{t+1} \omega^e \bar{K}_{j,t} dF^e(\omega^e) dj = u_{t+1} \bar{K}_t.$$

As explained above, all entrepreneurs choose the same leverage, default cutoff and utilization. To prevent entrepreneurs from accumulating net worth to the point where

¹²This shock captures both changes in government expenditures and changes in net exports.

they are completely self-financed, we require that they pay a fixed dividend δ^e each period to patient households. We also include an equity shock γ_t^e that shifts their aggregate net worth. Aggregate net worth after dividend payments is

$$N_t^e = \gamma_t^e [1 - \Gamma^e(\bar{\omega}_t^e)] R_t^k Q_{t-1}^k \bar{K}_{t-1} - \delta^e N_t^e.$$

Banks.—The aggregate balance sheet of the banking sector is

$$B_t \equiv B_t^i + B_t^e = D_t.$$

B.7 Summary of Equilibrium Conditions

We stationarize our model by defining the following scaled variables

$$\begin{split} b_t^e &= B_t^e / (z_t^* P_t), & h_t = \bar{H}_t / z_t^*, & n_t^i = N_t^i / (z_t^* P_t), & w_t^i = W_t^i / (z_t^* P_t), \\ b_t^i &= B_t^i / (z_t^* P_t), & h_t^i = \bar{H}_t^i / z_t^*, & q_t^h = Q_t^h / P_t, & w_t^p = W_t^p / (z_t^* P_t), \\ c_t &= C_t / z_t^*, & h_t^p = \bar{H}_t^p / z_t^*, & \tilde{q}_t^h = \tilde{Q}_t^h / P_t, & y_{z,t} = Y_t / z_t^*, \\ c_t^i &= C_t^i / z_t^*, & i_t = I_t / (z_t^* \Upsilon^t), & q_t^k = Q_t^k \Upsilon^t / P_t, & y_t = Y_t^{gdp} / z_t^*, \\ c_t^p &= C_t^p / z_t^*, & k_t = \bar{K}_t / (z_t^* \Upsilon^t), & \tilde{q}_t^k = \tilde{Q}_t^k \Upsilon^t / P_t, & \mu_{z^*,t} = z_t^* / z_{t-1}^*, \\ d_t &= D_t / z_t^*, & \lambda_t^i = \Lambda_t^i P_t z_t^*, & s_t = S_t / (z_t^* P_t), \\ F_{w,t}^i &= F_{W,t}^i z_t^*, & mc_t = MC_t / P_t, & t_t = T_t / (z_t^* P_t), \\ g_t &= G_t / z_t^*, & n_t^e = N_t^e / (z_t^* P_t), & t_t^i = T_t^i / (z_t^* P_t), \end{split}$$

Prices and wages

$$F_{p,t}^{p} = \lambda_{t}^{p} y_{z,t} + \xi_{p} \beta^{p} E_{t} (\tilde{\pi}_{t+1} \pi_{t+1}^{-1})^{\frac{1}{1-\lambda_{p,t+1}}} F_{p,t+1}^{p}.$$
(1)

$$K_{p,t}^{p} = \lambda_{t}^{p} y_{z,t} \lambda_{p,t} m c_{t} + \xi_{p} \beta^{p} E_{t} (\tilde{\pi}_{t+1} \pi_{t+1}^{-1})^{\frac{\lambda_{p,t+1}}{1-\lambda_{p,t+1}}} K_{p,t+1}^{p}.$$
(2)

$$K_{p,t}^{p} = \left(\left[1 - \xi_{p} (\tilde{\pi}_{t} \pi_{t}^{-1})^{\frac{1}{1 - \lambda_{p,t}}} \right] (1 - \xi_{p})^{-1} \right]^{1 - \lambda_{p,t}} F_{p,t}^{p}.$$
(3)

$$F_{w,t}^{p} = (1 - \tau^{l})\lambda_{w}^{-1}\lambda_{t}^{p}l_{t}^{p} + \xi_{w}\beta^{p}\mu_{z^{*}}^{\frac{1}{1-\lambda_{w}}}E_{t}\mu_{z^{*,t+1}}^{-1}\pi_{w,t+1}^{\frac{\lambda_{w}}{\lambda_{w}-1}}\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_{w}}}\pi_{t+1}^{-1}F_{w,t+1}^{p}.$$
(4)

$$K_{w,t}^{p} = l_{t}^{p,1+\sigma_{l}} + \xi_{w}\beta^{p}E_{t}(\tilde{\pi}_{w,t+1}\pi_{w,t+1}^{-1}\mu_{z^{*}})^{\frac{A_{w}}{1-A_{w}}(1+\sigma_{l})}K_{w,t+1}^{p}.$$
(5)

$$K_{w,t}^{p} = \psi_{l}^{-1} \left[\left(1 - \xi_{w} (\tilde{\pi}_{w,t} \pi_{w,t}^{-1} \mu_{z^{*}})^{\frac{1}{1 - \lambda_{w}}} \right) (1 - \xi_{w})^{-1} \right]^{1 - \lambda_{w} (1 + \sigma_{l})} w_{t}^{p} F_{w,t}^{p}.$$
(6)

$$F_{w,t}^{i} = (1 - \tau^{l})\lambda_{w}^{-1}\lambda_{t}^{i}l_{t}^{i} + \xi_{w}\beta^{i}\mu_{z^{*}}^{\frac{1}{1-\lambda_{w}}}E_{t}\mu_{z^{*,t+1}}^{-1}\pi_{w,t+1}^{\frac{\lambda_{w}}{\lambda_{w-1}}}\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_{w}}}\pi_{t+1}^{-1}F_{w,t+1}^{i}.$$
(7)

$$K_{w,t}^{i} = l_{t}^{i,1+\sigma_{l}} + \xi_{w}\beta^{i}E_{t}(\tilde{\pi}_{w,t+1}\pi_{w,t+1}^{-1}\mu_{z^{*}})^{\frac{A_{w}}{1-A_{w}}(1+\sigma_{l})}K_{w,t+1}^{i}.$$
(8)

$$K_{w,t}^{i} = \psi_{l}^{-1} \left[\left(1 - \xi_{w} (\tilde{\pi}_{w,t} \pi_{w,t}^{-1} \mu_{z^{*}})^{\frac{1}{1 - \lambda_{w}}} \right) (1 - \xi_{w})^{-1} \right]^{1 - \lambda_{w} (1 + \sigma_{l})} w_{t}^{i} F_{w,t}^{i}.$$
(9)

Production, resource constraints, and government

$$r_t^k = r^k \exp(\sigma_a[u_t - 1]).$$
(10)

$$r_t^k = \alpha \varepsilon_t (\Upsilon \mu_{z^*, t} l_t)^{1-\alpha} (u_t k_{t-1})^{\alpha - 1} m c_t.$$

$$\tag{11}$$

$$w_t^p = (1 - \alpha) \kappa m c_t \varepsilon_t \Upsilon^{-\alpha} (\mu_{z^*, t}^{-1} u_t k_{t-1})^{\alpha} l_t^{1-\alpha} / l_t^p.$$

$$\tag{12}$$

$$w_t^i = (1 - \alpha)(1 - \kappa)mc_t \varepsilon_t \Upsilon^{-\alpha} (\mu_{z^*, t}^{-1} u_t k_{t-1})^{\alpha} l_t^{1-\alpha} / l_t^i.$$
(13)

$$k_t = (1 - \delta)\Upsilon^{-1} \mu_{z^*, t}^{-1} k_{t-1} + [1 - S^k \left(\Upsilon \mu_{z^*, t} \zeta_{i, t} i_t / i_{t-1}\right)] i_t.$$
⁽¹⁴⁾

$$R_t^k = \left[(1 - \tau^k) [u_t r_t^k - a(u_t)] + (1 - \delta) q_t^k \right] \Upsilon^{-1} \pi_t q_{t-1}^{k, -1} + \tau^k \delta.$$
(15)

$$R_t^h = \pi_t q_t^h / q_{t-1}^h.$$
(16)

$$y_{z,t} = \varepsilon_t (\Upsilon^{-1} \mu_{z^*,t}^{-1} u_t k_{t-1})^{\alpha} l_t^{1-\alpha} - \theta.$$
(17)

$$y_{z,t} = g_t + c_t + \mu_{\Upsilon,t}^{-1} i_t + a(u_t) \Upsilon^{-1} \mu_{z^*,t}^{-1} k_{t-1}.$$
(18)

$$c_t = c_t^p + c_t^1. \tag{19}$$

$$l_t = l_t^{p,\kappa} l_t^{l,1-\kappa}.$$
 (20)

$$h = h_t^p + h_t^i. aga{21}$$

$$y_t = g_t + c_t + \mu_{\Upsilon,t}^{-1} i_t.$$
(22)

$$R_{t} = R + \rho_{p}(R_{t-1} - R) + (1 - \rho_{p}) \left[\alpha_{\pi}(E_{t}\pi_{t+1} - \pi) + \alpha_{\Delta y}(g_{y,t} - \mu_{z^{*}}) \right] + \varepsilon_{t}^{p},$$
(23)

$$g_{t} = \tau^{k} \left([u_{t}r_{t}^{k} - a(u_{t})]\Upsilon^{-1} - \pi_{t}^{-1}\delta q_{t-1}^{k} \right) \mu_{z^{*},t}^{-1} k_{t-1} + \tau^{l} (w_{t}^{i}l_{t}^{i} + w_{t}^{p}l_{t}^{p}) + \tau^{c}c_{t} - t_{t}.$$
(24)

$$0 = \lambda_t^p q_t^k \left[1 - S^k \left(\Upsilon \mu_{z^*, t} \zeta_{i, t} i_t / i_{t-1} \right) - \Upsilon \mu_{z^*, t} \zeta_{i, t} i_t / i_{t-1} S^{k'} \left(\Upsilon \mu_{z^*, t} \zeta_{i, t} i_t / i_{t-1} \right) \right]$$
(25)

$$-\mu_{\Upsilon,t}^{-1}\lambda_t^p + \beta^p E_t(\Upsilon\mu_{z^*,t+1})^{-1}\lambda_{t+1}^p q_{t+1}^k \zeta_{i,t+1} \left(\Upsilon\mu_{z^*,t+1}i_{t+1}/i_t\right)^2 S^{k'}\left(\Upsilon\mu_{z^*,t+1}\zeta_{i,t+1}i_{t+1}/i_t\right).$$

Patient households

$$0 = (1 + \tau^c)\lambda_t^p - \mu_{z^*,t}\zeta_{c,t}/(\mu_{z^*,t}c_t^p - b_c^p c_{t-1}^p) + b_c^p \beta^p E_t \zeta_{c,t+1}/(\mu_{z^*,t+1}c_{t+1}^p - b_c^p c_t^p).$$
(26)

$$0 = 1/h_t^p - \lambda_t^p q_t^h + \beta^p E_t \mu_{z^*,t+1}^{-1} \lambda_{t+1}^p q_{t+1}^h.$$
(27)

$$0 = \lambda_t^p - \beta^p E_t (\pi_{t+1} \mu_{z^*, t+1})^{-1} \lambda_{t+1}^p R_t.$$
(28)

$$0 = \lambda_t^p - \beta^p E_t (\pi_{t+1} \mu_{z^*, t+1})^{-1} \lambda_{t+1}^p R_t^s / \nu_t.$$
⁽²⁹⁾

Impatient households

$$0 = (1 + \tau^{c})\lambda_{t}^{i} - \mu_{z^{*,t}}\zeta_{c,t}/(\mu_{z^{*,t}}c_{t}^{i} - b_{c}^{i}c_{t-1}^{i}) + b_{c}^{i}\beta^{i}E_{t}\zeta_{c,t+1}/(\mu_{z^{*,t+1}}c_{t+1}^{i} - b_{c}^{i}c_{t}^{i}).$$
(30)

$$0 = 1 + \lambda_{t}^{i}b_{t}^{i} - \lambda_{t}^{i}q_{t}^{h}h_{t}^{i}[1 + S^{h}(\mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i}) + \mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}h_{t-1}^{i-1}S^{h'}(\mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i})]$$

$$+ \rho_{t}^{i}E_{t}\omega_{t}^{-1} - \rho_{t}^{i}\omega_{t}^{h}h_{t}^{i}[1 + S^{h}(\mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i}) + \mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}h_{t-1}^{i-1}S^{h'}(\mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i})]$$

$$+ \rho_{t}^{i}E_{t}\omega_{t}^{-1} - \rho_{t}^{i}\omega_{t}^{h}h_{t}^{i}[1 + S^{h}(\mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i}) + \mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}h_{t-1}^{i-1}S^{h'}(\mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i})]$$

$$+ \rho_{t}^{i}E_{t}\omega_{t}^{-1} - \rho_{t}^{i}\omega_{t}^{h}h_{t}^{i}[1 + S^{h}(\mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i}) + \mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i})]$$

$$+ \rho_{t}^{i}E_{t}\omega_{t}^{-1} - \rho_{t}^{i}\omega_{t}^{h}h_{t}^{i}[1 + S^{h}(\mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i}) + \mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i})]$$

$$+ \rho_{t}^{i}E_{t}\omega_{t}^{-1} - \rho_{t}^{i}\omega_{t}^{h}h_{t}^{i}[1 + S^{h}(\mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i}) + \rho_{t}^{i}\omega_{t}^{h}h_{t}^{i}[1 + S^{h}(\mu_{z^{*,t}}\zeta_{h,t}h_{t}^{i}/h_{t-1}^{i}) + \rho_{t}^{i}\omega_{t}^{h}h_{t}^{i}/h_{t-1}^{i})]$$

$$+ \beta^{i} E_{t} \mu_{z^{*},t+1}^{-1} \lambda_{t+1}^{i} q_{t+1}^{h} h_{t}^{i} \zeta_{h,t+1} (\mu_{z^{*},t+1} h_{t+1}^{i} / h_{t}^{i})^{2} S^{h\prime} (\mu_{z^{*},t+1} \zeta_{h,t} h_{t+1}^{i} / h_{t}^{i})$$

$$+ \beta^{i} E_{t} (\pi_{t+1} \mu_{z^{*},t+1})^{-1} \lambda_{t+1}^{i} [1 - \Gamma^{i} (\bar{\omega}_{t+1}^{i})] R_{t+1}^{h} q_{t}^{h} h_{t}^{i}.$$

$$(31)$$

$$0 = \lambda_t^i - \beta^i E_t (\pi_{t+1} \mu_{z^*, t+1})^{-1} \lambda_{t+1}^i [R_t + F^{i\prime}(\bar{\omega}_{t+1}^i) R_{t+1}^i (\bar{\omega}_{t+1}^i - \pi_{t+1} \tilde{Q}_{t+1}^h / [R_{t+1}^h Q_t^h])].$$
(32)

$$0 = (1 - \tau^{l})w_{t}^{i}l_{t}^{i} + (\pi_{t}\mu_{z^{*,t}})^{-1}[1 - \Gamma^{i}(\bar{\omega}_{t}^{i})]R_{t}^{h}q_{t-1}^{h}h_{t-1}^{i} + b_{t}^{i} + t_{t}^{i} - (1 + \tau^{c})c_{t}^{i} - q_{t}^{h}h_{t}^{i}.$$
 (33)

$$0 = R_{t-1}b_{t-1}^{i} - [1 - F^{i}(\bar{\omega}_{t}^{i})]R_{t}^{i}b_{t-1}^{i} - F^{i}(\bar{\omega}_{t}^{i})\pi_{t}\tilde{q}_{t}^{n}h_{t-1}^{i}.$$
(34)

$$\bar{\omega}_t^i = R_t^i b_{t-1}^i / (R_t^h q_{t-1}^h h_{t-1}^i).$$
(35)

$$n_t^i = (\pi_t \mu_{z^*,t})^{-1} [1 - \Gamma^i(\bar{\omega}_t^i)] R_t^h q_{t-1}^h h_{t-1}^i.$$
(36)

Entrepreneurs

$$0 = E_t [1 - \Gamma_1^e(\bar{\omega}_{t+1}^e)] \frac{R_{t+1}^k}{R_t} - \frac{E_t \Gamma_1^{e'}(\bar{\omega}_{t+1}^e)}{\Gamma_2^{e'}(\bar{\omega}_{t+1}^e) + [\pi_{t+1}\tilde{q}_{t+1}^k/(\Upsilon R_{t+1}^k q_t^k) - 1] F^{e'}(\bar{\omega}_{t+1}^e)} \frac{1}{L_t^e}.$$
 (37)

$$0 = R_{t-1}b_{t-1}^{e} - [1 - F^{e}(\bar{\omega}_{t}^{e})]R_{t}^{e}b_{t-1}^{e} - F^{e}(\bar{\omega}_{t}^{e})\Upsilon^{-1}\pi_{t}\tilde{q}_{t}^{k}k_{t-1}.$$

$$(38)$$

$$\bar{\omega}_{t}^{e} = R_{t}^{e}b_{t-1}^{e}/(R_{t}^{k}q_{t-1}^{k}k_{t-1}).$$

$$(39)$$

$$\bar{\omega}_t^e = R_t^e b_{t-1}^e / (R_t^k q_{t-1}^k k_{t-1}).$$
(39)

$$n_t^e = \gamma_t^e (\pi_t \mu_{z^*, t})^{-1} [1 - \Gamma^e(\bar{\omega}_t^e)] R_t^k q_{t-1}^k k_{t-1} - \delta^e n_t^e.$$
(40)

$$L_t^e = q_t^k k_t / n_t^e. aga{41}$$

$$b_t^e = q_t^k k_t - n_t^e. aga{42}$$

Financial sector

$$s_{t} = F^{i}(\bar{\omega}_{t}^{i})\mu_{z^{*},t}^{-1}\tilde{q}_{t}^{h}h_{t-1}^{i} + F^{e}(\bar{\omega}_{t}^{e})\Upsilon^{-1}\mu_{z^{*},t}^{-1}\tilde{q}_{t}^{k}k_{t-1}.$$
(43)

$$0 = (1 - \mu^{h})G^{i}(\bar{\omega}_{t}^{i})R_{t}^{h}q_{t-1}^{h}h_{t-1}^{i} + (1 - \mu^{k})G^{e}(\bar{\omega}_{t}^{e})R_{t}^{k}q_{t-1}^{k}k_{t-1} - \pi_{t}\mu_{z^{*},t}R_{t}^{s}s_{t}.$$
 (44)

$$0 = (1 - \mu^{h}) R_{t}^{h} \frac{q_{t-1}^{n}}{\tilde{q}_{t}^{h}} \frac{G^{i}(\omega_{t}^{i})}{F^{i}(\bar{\omega}_{t}^{i})} - (1 - \mu^{k}) \Upsilon R_{t}^{k} \frac{q_{t-1}^{k}}{\tilde{q}_{t}^{k}} \frac{G^{e}(\omega_{t}^{e})}{F^{e}(\bar{\omega}_{t}^{e})}.$$
(45)

C Data

Our macroeconomic data comes from Eurostat, the European Comission, and Cepremap, a research center in France. Our financial data is provided by the European Central Bank. Table 3 lists all the variables.

Raw Series	Code	Source
Gross domestic product	GDP	Eurostat
Gross domestic product: price deflator	GDPDEF	Eurostat
Consumption excluding durable goods	CND	Eurostat
Consumption of durable goods	CD	Eurostat
Gross capital formation	GCF	Eurostat
Total hours worked	HOURS	Eurostat
Population	POP	Eurostat
Three-month money market interest rate	NOM	Eurostat
Credit to households	СН	European Central Bank
Credit to non-financial corporations	CNFC	European Central Bank
Interest rate on loans for house purchase	IRLH	European Central Bank
Interest rate on loans to corporations	IRLC	European Central Bank
Residential property prices	HP	European Central Bank
Gross capital formation: price deflator	GCFDEF	Eurostat
Share of labor compensation in GDP	LS	European Commission
Effective consumption tax rate	TAUC	Cepremap
Effective labor income tax rate	TAUL	Cepremap
Effective corporate income tax	TAUK	Cepremap
Constructed Series		Formula
GDP		$Y = GDP/(GDPDEF \times POP)$
Consumption		$C = CND/(GDPDEF \times POP)$
Investment		$I = (CD + GCF)/(GDPDEF \times POP)$
Hours worked		L = HOURS/POP
Inflation		$\pi = \ln(\text{GDPDEF}) - \ln(\text{GDPDEF}_{-1})$
Nominal interest rate		R = NOM/4
Household credit		$B^{i} = CH/(GDPDEF \times POP)$
Firm credit		$B^{e} = CNFC/(GDPDEF \times POP)$
Household spread		$s^i = (IRLH - NOM)/4$
Firm spread		$s^e = (irlc - nom)/4$
House price		$Q^{h} = HP/GDPEF$

Table 3: Data Description

Consumption includes services and nondurable goods. Investment is defined as gross capital formation plus durable goods. We express GDP, consumption, investment, and the two credit series in real, per capita terms and take the logarithmic first difference. Hours are in per capita log difference. We measure inflation, the nominal rate, and the two credit spreads in level. House prices are in real terms and log difference. We demean all variables to prevent low-frequency movements from interfering with the higher business-cycle frequencies that interest us.

Calibrated Parameters		Target / Source		Value
Capital share in production	α	Sample mean		0.3687
Government spending to GDP	η_g	Sample mean		0.2384
Inflation, annual %	π	Sample mean		1.4986
Per capita GDP growth, annual %	μ_{z^*}	Sample mean		1.0144
Investment price trend, annual %	Υ	Sample mean		0.7822
Tax rate on consumption	$ au^c$	Sample mean		0.1934
Tax rate on labor income	$ au^l$	Sample mean		0.1184
Tax rate on capital income	$ au^k$	Sample mean		0.3318
Patient discount factor	β^p	R = 1.87% 0.9		
Impatient discount factor	β^i	$\beta^i < \beta^p \qquad \qquad 0.9$		
Depreciation rate of capital	δ	10% annual 0.0		
Labor supply elasticity	σ_l	Literature		1.0000
Price markup	λ_p	Literature		1.2000
Wage markup	λ_w	Literature		1.0500
Disutility weight on labor	ψ_l	Hours $l = 1$		0.9576
Estimated Parameters		Prior(Mean,Std)	Mode	Standard Dev.
Taylor rule ouput	<i>a</i> .	N(0 5 0 05)	0 3080	0 3817
Taylor rule inflation	$u_{\Delta y}$	N(0.5, 0.05) N(1.5, 0.2)	0.5080	0.3617
Taylor rule smoothing	u_{π}	R(1.3, 0.2) R(0.75, 0.1)	2.1639	0.3031
Calvo price stickiness	ρ_p	B(0.75, 0.1) B(0.75, 0.15)	0.0033	0.3237
Calvo wage stickiness	$\frac{Sp}{c}$	B(0.75, 0.15) B(0.75, 0.15)	0.8270	0.0210
Price indevation on inflation	Sw	B(0.75, 0.15) B(0.75, 0.1)	0.7082	0.0203
Wage indexation on inflation	ι_p	B(0.75, 0.1) B(0.75, 0.1)	0.9349	0.1021
Patient consumption habit	l_W h^p	B(0.75, 0.1) B(0.6, 0, 1)	0.8195	0.2034
Impatient consumption habit	b_c^i	B(0.6, 0.1)	0.7534	0.0040
Capital utilization cost	v_c	N(1, 0, 25)	1 /372	0.0520
Investment adjustment cost	$S_{a}^{k''}$	N(1, 0.23) N(2, 0, 5)	2 4513	0.2051
Housing adjustment cost	S Shii	N(2, 0.3) N(2, 20)	2.4313	10,6262
Share of nations in total labor	3	P(0.5, 0.1)	0 7450	0 1603
Imposion default probability	K $E^{i}(-i)$	B(0.3, 0.1) B(0.007, 0.003)	0.7430	0.1003
Entropropour default probability	$F(\omega)$ $F^{e}(-e)$	B(0.007, 0.003) B(0.007, 0.003)	0.0130	0.0040
Entrepreneur leverage	$F(\omega)$	B(0.007, 0.005) N(2, 1, 0, 2)	0.0078	0.0013
Markup in socuritized housing	L h	P(0.4, 0.15)	0.1018	0.4425
Markup in securitized nousing	μ μ	B(0.4, 0.15)	0.1916	0.0710
Markup in securitized capital	μ	B(0.4, 0.13)	0.2219	0.1407
Bank shock	ρ_{ν}, σ_{ν}	B(.5, .2), G(.01, 2)	0.939, 0.073	0.041, 0.0298
Stationary technology shock	$\rho_{\varepsilon}, \sigma_{\varepsilon}$	B(.5, .2), G(.01, 2)	0.928, 0.003	0.042, 0.0003
Permanent technology shock	$ ho_{\mu_{z^*}}, \sigma_{\mu_{z^*}}$	B(.5, .2), G(.01, 2)	0.855, 0.002	0.112, 0.0014
Stationary investment shock	$\rho_{\zeta_I}, \sigma_{\zeta_I}$	B(.5,.2), G(.01,2)	0.311, 0.015	0.232, 0.0034
Permanent investment shock $\rho_{\mu_{\Upsilon}}, \sigma_{\mu_{\Upsilon}}$		B(.5, .2), G(.01, 2)	0.925, 0.007	0.047, 0.0023
Preference shock $\rho_{\zeta_c}, \sigma_{\zeta_c}$		B(.5, .2), G(.01, 2)	0.394, 0.014	0.516, 0.0061
Housing shock $\rho_{\zeta_h}, \sigma_{\zeta_h}$		B(.5, .2), G(.01, 2)	0.812, 0.002	0.070, 0.0004
Markup shock ρ_{λ_p}		B(.5, .2), G(.01, 2)	0.042, 0.067	0.150, 0.0275
Firm equity shock $\rho_{\gamma^e}, \sigma_{\gamma^e}$		B(.5, .2), G(.01, 2)	0.435, 0.003	0.518, 0.0020
Firm risk shock ρ_{σ^e}, σ		B(.5, .2), G(.01, 2)	0.952, 0.005	0.048, 0.0097
Government spending shock ρ_g, σ_g		B(.5, .2), G(.01, 2)	0.896, 0.012	0.058, 0.0013
Monetary policy shock σ_{ε^p}		G(.01,2)	0.0004	0.0007

Table 4: Parameters

Note: N, B, and G stand for normal, beta, and inverse gamma distribution, respectively.

D Estimation

This section contains details about the model's estimation. We discuss the parameterization and present two measures of model fit.

D.1 Calibrated Parameters

A number of model parameters have a clear counterpart in the data, so we calibrate them to match the mean in our sample. These include the annual growth rate of the economy $\mu_{z^*} = 1.01\%$; the annual inflation rate $\pi = 1.50\%$; the annual rate of investment-specific technological change $\Upsilon = 0.78\%$; the share of government spending in GDP $\eta^g = 0.24$; the capital share in production $\alpha = 0.37$; and the tax rates on consumption $\tau^c = 0.19$, labor income $\tau^l = 0.12$, and capital income $\tau^k = 0.33$.

We fix a few other parameters as follows. The discount factor of patient households β^p equals 0.9998, which pins down the annualized nominal interest rate *R* to 1.87%. The discount factor of impatient households β^i must be lower than β^p and is set to 0.97. Capital depreciation δ and labor supply elasticity σ_L are fixed at 0.025 and 1, respectively. We set the steady-state price and wage markups λ_p and λ_w to 1.20 and 1.05, following the literature. All calibrated parameters appear at the top of Table 4.

D.2 Estimated Parameters

We estimate the remaining 41 parameters with Bayesian techniques. The bottom panel of Table 4 reports their prior and posterior densities. Most of the structural parameters are common in the literature and are assigned standard priors.¹³ Our results fall in line with previous studies. For instance, the policy response to inflation α_{π} is 2.19 while the interest rate smoothing coefficient ρ_p is 0.66, consistent with existing estimates. Posterior values for the Calvo price ξ_p and wage ξ_w stickiness, of 0.83 and 0.77 respectively, are sensible compared to 0.91 and 0.74 found by Smets and Wouters (2003). Consumption habit of patient b_c^p and impatient households b_c^i , at 0.82 and 0.75, look reasonable. One exception is the price indexation parameter ι_p . Over the period from 1999Q1 to 2019Q4 covered by our sample, inflation in the eurozone was remarkably low and stable. The estimation procedure accommodates this by ascribing a high value of 0.93 to this coefficient.

We now discuss the less habitual parameters. The cost of adjusting housing $S^{h''}$ is essential to smooth the dynamics of impatient household housing, and hence household debt, a variable we observe. Because it is costly to dispose of housing immediately, impatient households react gradually to shocks. The posterior mode of $S^{h''}$, at 52, is

¹³The main references are Smets and Wouters (2003, 2007), Justiniano, Primiceri, and Tambalotti (2010), and Christiano, Motto, and Rostagno (2014).

what it takes to discipline the dynamics of household debt. Another important parameter is the share κ of patient households in the economy. We set its prior to 0.5 based on the observation that at least half of households in Europe hold some form of debt. Our posterior estimate is 0.74, suggesting financially-constrained consumers represent roughly a quarter of the population, in accord with micro evidence in Kaplan, Violante, and Weidner (2014). Next, we set the prior mean of the steady-state default probability of households $F^i(\bar{\omega}^i)$ and entrepreneurs $F^e(\bar{\omega}^e)$ to an annual percentage rate of 3. We find a higher estimate for $F^i(\bar{\omega}^i)$, consistent with the fact that households default more than firms.¹⁴ Finally, two parameters μ^h and μ^k are specific to the financial sector. They govern the markup of shadow banks in the securitized mortgage and business loan markets, respectively. We center their prior around a mean of 40 percent, in the upper range of data on pre-tax return on equity. Posterior estimates are lower, at 0.19 for μ^h and 0.22 for μ^k , indicating the model does not need a high degree of financial frictions to perform well quantitatively.

We turn to the exogenous processes. Each row in the lower part of Table 4 corresponds to a shock and reports a pair (ρ, σ). The only exception is the monetary policy shock whose autocorrelation we set to zero, given that there is already a smoothing parameter in the Taylor rule. The market sentiment shock stands out, with the highest standard deviation of all shocks, 0.07, and a persistence of 0.939. This autocorrelation is lower than what the literature usually finds for financial and uncertainty shocks, typically 0.95–0.98, and is one reason why the responses of our model to a market sentiment shock are close to those of a less structural VAR.

D.3 Model Fit

We look at the steady-state properties of our stylized economy as a first measure of model performance. The top panel of Table 5 reports selected ratios and variables when parameters are set to their posterior mode, along with the analog objects in the data. The model and data match well. This is the case by construction for the ratio of government spending to GDP and inflation. The good fit is not trivial because our observables are demeaned and expressed mostly in growth rate and therefore the estimation procedure does not use information in the data about the sort of ratios shown in Table 5. An exception to the good match is the nominal interest rate, too high in the model. Given that in steady state $R = \pi \mu_{z^*}/\beta^p$ and $\pi > 0$ and $\mu_{z^*} > 0$, we reach a lower bound for R as we increase β^p closer to one.

Our second measure of model fit relates to the dynamic properties of our economy.

¹⁴We treat steady-state entrepreneurial leverage L^e as a parameter and estimate it, setting its prior mean to the average of 2.1 in our sample. We then calibrate the entrepreneurial dividend parameter δ^e to 0.018 so as to be consistent with L^e 's posterior mode of 1.57.

Steady-State Variables					Model	Data		
Consumption to GDP c/y					0.55	0.49		
Investment to GDP	i/y		0.22	0.27				
Government spending t	g/y		0.24	0.24				
Debt to GDP	b/(4y)		1.38	1.45				
Household debt to total	b^i/b		0.48	0.40				
Inflation, annual rate π					1.51	1.51		
Nominal interest rate, a	nnual		R		2.61	1.85		
Business sector leverage L^e					1.57	2.10		
	-							
Dynamic Variables	Correlation		Standard Deviation		Autocorrelation			
	Model	Data	Model	Data	Model	Data		
GDP	1.00	1.00	1.00	1.00	0.94	0.92		
Consumption	0.98	0.92	0.67	0.76	0.94	0.92		
Investment	0.99	0.97	2.97	2.92	0.95	0.91		
Hours	0.99	0.91	1.27	0.59	0.94	0.92		
Inflation	0.70	0.77	0.12	0.08	0.91	0.91		
Nominal rate	0.86	0.81	0.25	0.58	0.93	0.92		
Household credit	0.55	0.65	0.92	0.51	0.96	0.96		
Business credit	0.46	0.13	1.09	0.79	0.97	0.92		
Household spread	-0.69	-0.91	0.10	0.46	0.90	0.91		
Business spread	-0.53	-0.96	0.07	0.27	0.90	0.90		

Table 5: Static and Dynamic Properties, Model Versus Data

Notes: Model values are computed at the posterior mode. In the bottom panel model and data variables are detrended with a bandpass filter (6,32). Standard deviations are normalized to that of GDP.

We simulate the model by shutting down all shocks except the market sentiment shock. The bottom panel of Table 5 reports moments of selected variables, where both the artificial and actual data are filtered with a bandpass filter (6,32). By and large, the model hit by a unique disturbance does a good job at replicating the salient features of the data. The main shortcomings are the model's volatility of hours, too high, and interest rates, too low. We believe there are institutional arrangements in the European labor market that limit fluctuations in employment. Still, it is remarkable that the model matches the data at business-cycle frequency so closely, given that it was not estimated in the frequency domain.