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Real and nominal effects of monetary shocks under time-varying disagreement

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Abstract

How do varying degrees of information frictions affect the transmission mechanism of monetary policy? Using non-linear methods, I empirically find that during heightened disagreement, monetary policy has a smaller effect on inflation, yet more influence over output. As a proxy for information frictions, I use real GDP nowcast disagreement across professional forecasters. Significant nowcast disagreement indicates when it is difficult to observe the current economic state, or a period of high information rigidities. I develop a tractable theoretical model that shows rationally inattentive price-setters produce this result. Improved central bank communication that reduces disagreement among economic agents can mitigate output losses when implementing disinflationary monetary policies.

Key words: Time-varying disagreement, monetary policy, state-dependent local projections, rational inattention.

JEL classification: E32, E52, E58, D83.

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1 Introduction

When studying the effects of monetary policy, standard macroeconomic models assume that all agents can perfectly observe the current state. However, existing works (see Woodford (2009), Andrade and Le Bihan (2013) and Andrade et al. (2016)) find there is a significant cost from processing information about the current economic state that generates a quantitatively important effect on monetary transmission (Sims, 2005; Song and Stern, 2022). This paper examines how varying degrees of information frictions affect the transmission mechanism of monetary policy.

The main empirical results find that using non-linear methods, during heightened information frictions, monetary policy has a *smaller* effect on inflation, yet *more* influence over output. As a proxy for information frictions, I use disagreement about nowcast of real GDP from the U.S. Survey of Professional Forecasters (SPF). Nowcast disagreement across economic agents makes for a particularly useful proxy for information frictions. A significant amount of disagreement on a near-term forecast indicates a period when it is difficult to observe and assess the current economic conditions – in other words, high information frictions. If agents' ability to nowcast varies over time, it may affect their ability to respond to various shocks, including monetary shocks. I construct a narratively-identified monetary policy shock series – by extending Romer and Romer (2004) and estimating the shocks non-linearly according to the disagreement regimes. Impulse responses to the narrative monetary policy shocks are estimated with local projections method.[1]

To illustrate why disagreement could be crucial for monetary policy, I design a tractable rational inattention model (where firms decide to optimally allocate their attention).[2] This model offers an explanation to the empirical results by examining how price-setting changes with varying information frictions and how it impacts monetary policymaking on central banks' goal variables. During high disagreement periods, price-setting firms pay less attention to demand conditions. The model suggests that when firms are only able to imperfectly observe factors that affect their optimal prices, they attach a positive (but less than unity) weight to the signals they receive (the 'Kalman gain') on these factors. This implies that their prices respond sluggishly to aggregate monetary shocks. The slower prices respond, the more 'sticky' prices appear, leading to a flatter Phillips curve. Thus, output would correspondingly react by more to a monetary policy shock.

[1]The empirical results also hold in an endogenous threshold VAR with recursive monetary policy shocks in Section 3, as well as threshold VAR with narrative monetary policy shocks in Appendix A.6.

[2]The tractable rational inattention model setup is closest to Zhang (2017). In her paper, the attention paid to a particular variable only depends on the prior uncertainty of the variable itself, and the aggregate marginal cost of attention. In contrast to Zhang but in line with Maćkowiak and Wiederholt (2009), as agents allocate a finite attention, the relative variance across different variables also matters.

This paper contributes to three strands of literature. First, it builds on the growing works of disagreement among economic agents (Andrade et al., 2016; Reis, 2020), and in particular on the empirical estimation of monetary policy transmission under disagreement. The literature has mostly focused on disagreement about the *forecast* of *inflation* (inflation expectations) (Mankiw et al., 2004; Coibion and Gorodnichenko, 2012, 2015; Falck et al., 2021). In the baseline exercise, I focus on *nowcast* disagreement because economic agents are not only forward-looking, but in reality, households and firms also try to infer the current state of the economy when making decisions. If professional forecasters instead disagree about forecast (instead of nowcast) of real GDP (or inflation), Section 4 finds the response of output and inflation to be well within the confidence interval of the main results under heightened disagreement. However, during low disagreement monetary policy shocks produce a contractionary effect, that is, output do not respond differently in the two regimes – replicating the findings in Falck et al. (2021).

Second, it relates to the literature on state-dependent effects of monetary policy, in particular, under uncertainty.[3] The novel insight from the model in this paper is that it dissects the relationship between disagreement and uncertainty – two fundamentally different concepts.[4] While there is a large literature on uncertainty, disagreement has received relatively less attention while possibly being more relevant in studying information effects. Section 5.1.2 highlights how the two concepts distinctly affect monetary transmission, and when there is a positive link between them (or when they break down).[5] The model shows that an increase in uncertainty about demand condition co-moves with disagreement when attention on aggregate demand condition is already relatively high, such that paying additional attention may result in a lower marginal benefit. Hence, firms do not re-allocate more attention to demand conditions, resulting in a rise in disagreement. On the other hand, when the allocated attention on demand is still relatively low, an *increase* in demand uncertainty raises the benefit to monitoring demand conditions. Firms could then optimally re-allocate much more attention to monitoring demand and *decreases* disagreement on the assessment of demand across different firms.

[3]There is a wider literature looking into the effects of monetary shock in recession and expansion. Tenreyro and Thwaites (2016) find strong evidence that the effects of monetary policy on real *and* nominal variables are less powerful in recessions. Castelnuovo and Pellegrino (2018) and Aastveit et al. (2017) also point to a weak impact of monetary policy shocks on real activity under high uncertainty – the period they relate with recessions.

[4]An established literature proxied uncertainty with the disagreement of individual forecasts in surveys (Bomberger, 1996). However, the contemporary literature considers uncertainty and disagreement as two fundamentally different concepts (Rich and Tracy, 2006; Boero et al., 2008; Lahiri and Sheng, 2010; Abel et al., 2016). Empirically, various measures of macroeconomic uncertainty and disagreement have positive, but weak, correlations (Kozeniauskas et al., 2018).

[5]Rational inattention models have previously been used to examine economic conditions under uncertainty, such as in Zhang (2017) and Acharya and Wee (2020).

Lastly, this paper contributes to the fast growing literature on *designing* rational inattention models to understand monetary policy transmission (amongst many, Woodford (2009), Sims (2010), Maćkowiak and Wiederholt (2009) and Maćkowiak and Wiederholt (2015)).[6] However, these mechanisms have not been utilised much to explain the empirical evidence of state-dependent monetary transmission. This paper narrows the gap in the literature by applying the mechanisms from rational inattention models to analyse the non-linear effects of monetary policy.

Policymakers have been long interested in examining the state-dependent effects of monetary policy on central banks' stabilisation goals.[7] These results shed light on how increased communication by monetary policymakers can affect their ability to deliver on their stabilisation objectives. Prediction on current real GDP informs economic agents of their current real income that helps form decision making, such as households' consumption plan or firms' production plan. The mechanism that explains the empirical results in this paper suggests that communicating aggregate *real* conditions can help central banks achieve their objectives. As improved communication helps economic agents form expectations about current and future conditions, this reduces the disagreement of agents and potentially lowers the sacrifice ratio. With disinflationary monetary policies, a policymaker could achieve the same fall in inflation with a smaller fall in output.

This paper is structured as follows: Section 2 describes the data, measurement of disagreement and empirical methodologies. Section 3 discusses the main results and Section 4 digs deeper into the empirical analysis. Section 5 presents the rational inattention model to interpret the empirical findings. Section 6 concludes and provides policy implications of the results.

2 Econometric Methodology

2.1 Data Description

The quarterly data of real GDP, GDP deflator, commodity price index and effective Federal Funds Rate is from the Federal Reserve Economic Data (FRED) database.[8]

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[6] Andrade et al. (2016) and Falck et al. (2021) show that their empirical observation is consistent with predictions from dispersed information models, while Mankiw et al. (2004), Andrade and Le Bihan (2013) and Coibion et al. (2018) use models to show that inattention is due to sticky information á la Mankiw and Reis (2002) and rational inattention á la Sims (2003).

[7] For example, in a speech, Vitor Constancio (Vice-President of the ECB, December 2014) says "Novel and effective nonlinear techniques allow us to gain a deeper and more nuanced understanding of highly policy-relevant issues. We believe that these methods will be further incorporated among the tools routinely used also by central bankers as valued sources of policy advice."

[8] The choice of these variables is standard in the empirical literature studying monetary policy transmission as noted by Christiano et al. (1996), Sims (1992), and Bernanke and Gertler (1995). I include a commodity price index to control for energy and food price shocks in the threshold VAR

Real GDP and GDP deflator are measures of economic activity and prices, sourced from the Bureau of Economic Analysis, and are seasonally adjusted. The sample period for the baseline empirical analysis with local projections is from 1970Q1 to 2013Q4. Digging deeper into the empirical results, Section 4 estimates a threshold VAR with a sample period from 1970Q1 to 2018Q4. From 2009Q1 to 2015Q3, Wu and Xia (2016) shadow rate replaces the effective Federal Funds Rates (FFR) to account for the zero lower bound (ZLB) and quantitative easing.[9] During these periods, the FFR was between 0 and 0.25 percent. Thus, the 'Wu-Xia shadow interest rate' captures the overall monetary policy stance better than the FFR on its own.

2.2 Measuring Disagreement

To analyse the state-dependent effects, I first define the state of the economy in the empirical analysis using a measure of disagreement amongst forecasters from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters (SPF).[10] Disagreement is the interquartile range of *real GDP* for the current quarter *(nowcast)*, divided by the median of the current quarter as a means of normalisation.[11]

SPF is one of the longest standing quarterly macroeconomic surveys that started in 1968.[12] The availability of historical data is useful as it covers a variety of episodes in U.S. macroeconomic history, including important economic events in the 1970s. The number of survey respondents varies overtime but on average, it approximately has 50 professional forecasters.

Professional forecasters are some of the most informed agents in the economy,

to capture supply side factors that may influence output and prices. This data is from the Bureau of Labour Statistics, and is originally not seasonally adjusted. I have seasonally adjusted commodity price index using the Census Bureau's X-13 ARIMA-SEATS, with near identical results.

[9] To overcome this issue, Wu and Xia (2016) propose a non-linear term structure model to construct a shadow interest rate that captures the effect of unconventional monetary policies on the overall stance of monetary policy. In response to the global financial crisis, the Federal Open Market Committee (FOMC) took drastic measures that took the FFR in to the effective lower bound from December 2008 to 2015, as they set the target range for the FFR at 0 to 25 basis points. Additionally, the Fed took unconventional measures, such as quantitative easing, to further ease credit conditions and lower long-term interest rates. Thus, after December 2008, the FFR is less likely to describe the monetary policy stance well. The 'Wu-Xia shadow interest rate' is updated only if the target range for the FFR is at or above 25 to 50. On December 16, 2015, the FOMC raised the target range for the FFR to 25 to 50 basis points.

[10] In line with the literature, I measure disagreement using interquartile range (difference between the 75^{th} percentile and the 25^{th} percentile of the projections in levels or growth at a point in time) that is widely used in the literature to ensure that any outliers do not unfairly influence the variable of interest – the measure of disagreement. This is similar to using standard deviation as a measure of disagreement. However, as Sill (2014) shows, the standard deviation in cross-sectional forecasts is clearly more volatile, though tracks the interquartile range measure fairly closely.

[11] Section 4 looks into disagreement in forecast (one-year ahead) of real GDP and inflation.

[12] At its current format, each forecaster provide the same set of baseline variables for the current quarter and up to four quarters ahead, as well as annualised values for the following 2 years for certain variables. SPF also asks special variables and special questions with different horizons.

thus SPF serves as a conservative benchmark for information frictions in their forecasts' cross-sectional variation. If there was an increase of information frictions that reduces a professional forecaster's ability to predict macroeconomic aggregates – despite all publicly available information and forecasting techniques – then, we could expect there would be higher information frictions among other economic agents, such as households and firms. As proposed by Carroll (2003), news may spread epidemiologically from experts to other agents.

Figure 1 plots the disagreement measure for the current quarter (nowcasts) real GDP.[13] High disagreement periods are defined as the periods where the disagreement variable is above a threshold. In the local projections estimation, the threshold is the median disagreement from 1970Q1-2013Q4 (solid red line). I also plot the estimated value of the threshold parameter (dash red line) that will be used in the threshold VAR in Section 4, with the sample period 1970Q1-2018Q4. The grey shaded areas indicate the NBER business cycle contraction dates.

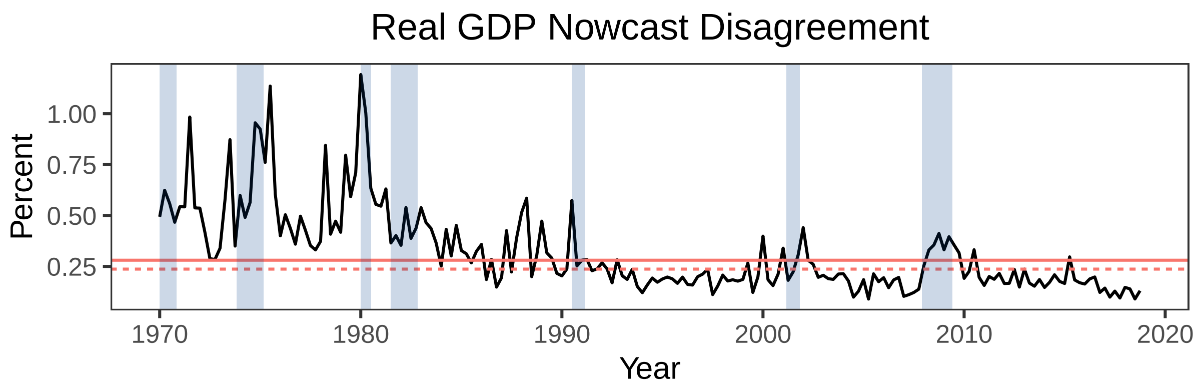

Figure 1. Time-Varying Real GDP Nowcast Disagreement

Note: Time series of the real GDP nowcast disagreement index based on the dispersion (interquartile range) of the U.S. SPF. The grey shaded areas indicate NBER-dated recessions. The solid red line indicates the state variable in the baseline local projections which is the median disagreement between 1970Q1-2013Q4. The dashed red line is the estimated threshold in the threshold VAR between 1970Q1-2018Q4. The y-axis is the interquartile range as a percentage of the median.

The chart shows that disagreement about current conditions is time-varying. Disagreement is higher in the early part of the sample period – I discuss below why this does not affect the empirical analysis. Time variation of survey dispersion has previously been observed in the literature for many different surveys, ranging from households, firms and professional forecasters, as well as for a variety of variables and a range of different forecast horizons, from nowcasts to 10-year ahead (Andrade et al., 2016). In line with this stylised fact, the dynamic of declining disagreement (in

[13]The SPF provides individual forecasts for the quarterly and annual level of chain-weighted real GDP. The dataset is seasonally adjusted. Prior to 1992, these are forecasts for real GNP.

SPF) is also observed by Ricco et al. (2016) in the disagreement about fiscal policy and Falck et al. (2021) in inflation expectations disagreement.

It is important to understand why information frictions as measured by disagreement is time varying. The intuition behind this stylised fact is that economic agents are not fully informed all the time and this naturally creates heterogeneity in beliefs that inherently changes over time. This observation is consistent with predictions arising from information frictions models. For example, in a sticky information model, some agents have more updated information sets than others that drives heterogeneity in beliefs across these agents at different times. In this paper, the rational inattention model in Section 5 provides a plausible explanation for the decline in disagreement in the latter half of the sample. Total attention toward macroeconomic conditions (K in the rational inattention model) may have increased as there is a generally greater effort in forecasting GDP and other macroeconomic variables in the past 30 years. During the sample period, forecasting methods and information available may have significantly improved. This would lower the noise in the signals, improving the signal-to-noise ratio and lowering the attention cost, and thus creates a lower disagreement among professional forecasters.

Furthermore, the change in the dynamic of the disagreement overtime cannot be pinned down to a particular time or event. Observing Figure 1, disagreement seems higher in the early years of the survey (pre-early 1990) in comparison with the latter half of the sample. This raises a question of whether there was a structural break in the SPF real GDP nowcasts in the early 1990s. An event that happened during this period is a change in the forecast variable from GNP to GDP.[14] However, this change was not unique to the survey. There was a consensus for the official measure of output to be GDP rather than GNP in 1991. The change from GNP to GDP can also be observed in other macroeconomic forecast surveys such as Blue Chips. In Figure 7 in Appendix A.1, real and nominal GNP tracks real and nominal GDP. As a check, a Wald structural break test points to 1980Q2 as the structural break in the disagreement variable, rather than the early 1990s.[15]

Moreover, it is unlikely that this pattern of declining disagreement can be fully attached to an economic event or policy regime. Although it tracks with the Great Moderation period from 1984 to 2008, notice that the fall in disagreement is not just a consequence of the Great Moderation. Even in this period when the overall volatility of the economic data was lower than in the pre-1984 period, we can still observe high

[14] Also in 1990, The Federal Reserve Bank of Philadelphia took over the survey. The survey began in 1968 and was conducted by the American Statistical Association and the National Bureau of Economic Research.

[15] The output of the Supremum Wald (test for a structural break at an unknown break date, with symmetric trimming of 15%) indicates to reject the null hypothesis of no structural break at the 5% level, with a test statistic of 157.4213.

disagreement particularly in the late 1980s to early 1990s, and around business cycle recession dates. While high disagreement is (weakly) correlated with recessions, high disagreement episodes are more prolonged after recessions, and disagreement regime changes typically occur at a higher frequency than business cycles. The correlation between the disagreement state variable in this paper and indicators of recession (or slack) is only mildly positive. The correlation with NBER-dated recessions is below 0.4. Additionally, similar to the observation in Falck et al. (2021), the variation of dovish and hawkish monetary policy conducts during periods of high and low disagreement makes it unlikely for the monetary policy regimes to be the main driver of variation in disagreement.

2.3 Non-Linear Narrative Shocks

The main analysis identifies monetary policy shocks with narrative identification approach that refers to the use of historical documents to reconstruct the intended policy target rate and the information set of policymakers. I extend the narrative identification by Romer and Romer (2004) (henceforth, RR) as the narrative monetary policy shocks, and estimate the shocks non-linearly similar to Tenreyro and Thwaites (2016). The non-linearly narratively identified monetary policy shocks will be applied to the local projections method.

Here, the effects of the monetary policy shocks are the residuals from an estimated reaction function. As in RR, I identify innovations to monetary policy by accounting for Federal Reserve's information set and follow their orthogonalisation procedure by regressing the Federal Funds target rate changes on Greenbook forecasts (and its revisions) at each FOMC meeting.[16] The original RR regression is:

$$\Delta FFR_t = \beta^b \mathbf{X}_t + \varepsilon_t \tag{1}$$

where $\mathbf{X}_t$ are the control variables employed by RR and the estimated residuals ε_t are the identified monetary policy shocks.[17] The state-dependent identification is:

$$\Delta FFR_t = F(z_{t-1})\beta^{(H)'}\mathbf{X}_t + (1 - F(z_{t-1}))\beta^{(L)'}\mathbf{X}_t + \tilde{\varepsilon}_t \tag{2}$$

where the estimated residuals $\tilde{\varepsilon}_t$ are the *non-linearly* identified monetary policy shocks. For exposition here, the dummy state variable is $F(z_t)$, where $F(z_t) = 1$ when in a high disagreement (H) state (defined as when z_t is above its median) and $F(z_t) = 0$

[16] In the zero lower bound periods, I regress the changes in the Wu and Xia (2016) shadow rate instead of the target rate. This is explained later in this section.

[17] The control variables are lags of Greenbook forecasts for GDP growth and GDP deflator, as well as their revisions since the last FOMC meeting. As with RR, I match the Greenbook used for the particular FOMC meeting.

when in a low disagreement (L) state.[18]

The original RR series provides narrative monetary policy shocks up to 1996. This paper extends the series up to 2013 and the literature has extended the series up to the financial crisis. I extend the series using the extended dataset provided by Wieland and Yang (2020) up to 2007, and to 2008 using Coibion et al. (2017). To complete the dataset to 2013, I hand-match the Greenbook forecasts, which are published with a five-year lag.

The state-dependent shocks have a 0.97 correlation with the RR original series over a common sample period. As the premise of this paper is that the behaviour of the economy is characterised by forms of state-dependent, it is possible that the FOMC's monetary policy reaction function may have also been state-dependent. In other words, estimating shocks with standard linear framework may include state-dependent measurement error (Tenreyro and Thwaites, 2016). Therefore, to account for this possibility, estimates of the narrative shock is analogous to the original RR but corresponding to the disagreement periods.

Romer and Romer (2004) notes that they analyse the Federal Reserve's intentions through the Federal Funds Rate (FFR) because for much of the sample the Federal Reserve targeted the FFR and the *change* in the intended FFR captures best what the Federal Reserve was aiming to do. The latter deals with the concern that in the sample period there were periods where the FOMC was not explicitly targeting the FFR. Therefore, using the change in the intended FFR serves as the easiest indicator of FOMC's intentions to deduce accurately over a long period of time and over a variety of monetary regimes. To maintain consistency, I also use this approach after the financial crisis. However, where the FFR has been near the zero lower bound, the target FFR would give zero variation in this period. In addition, the target FFR does not capture the true monetary policy stance, due to the use of unconventional monetary policies, such as quantitative easing (QE) and forward guidance. As a solution, for this period, Wu and Xia (2016) shadow rate replaces FFR in order to capture the additional features of unconventional monetary policy that have noticeable impact on the macroeconomy (Ramey, 2016).

[18] In Appendix A.5, I also define a smooth transition state using a logistic transformation as in Tenreyro and Thwaites (2016). This is then used for the smooth transition local projections. For consistency (across the baseline and the smooth transition local projection) in this section, $F(z_t)$ refers to the dummy state variable. Note that $F(z_t) = 1$ is equal to $I_t = 1$ in Eq (5) and (3).

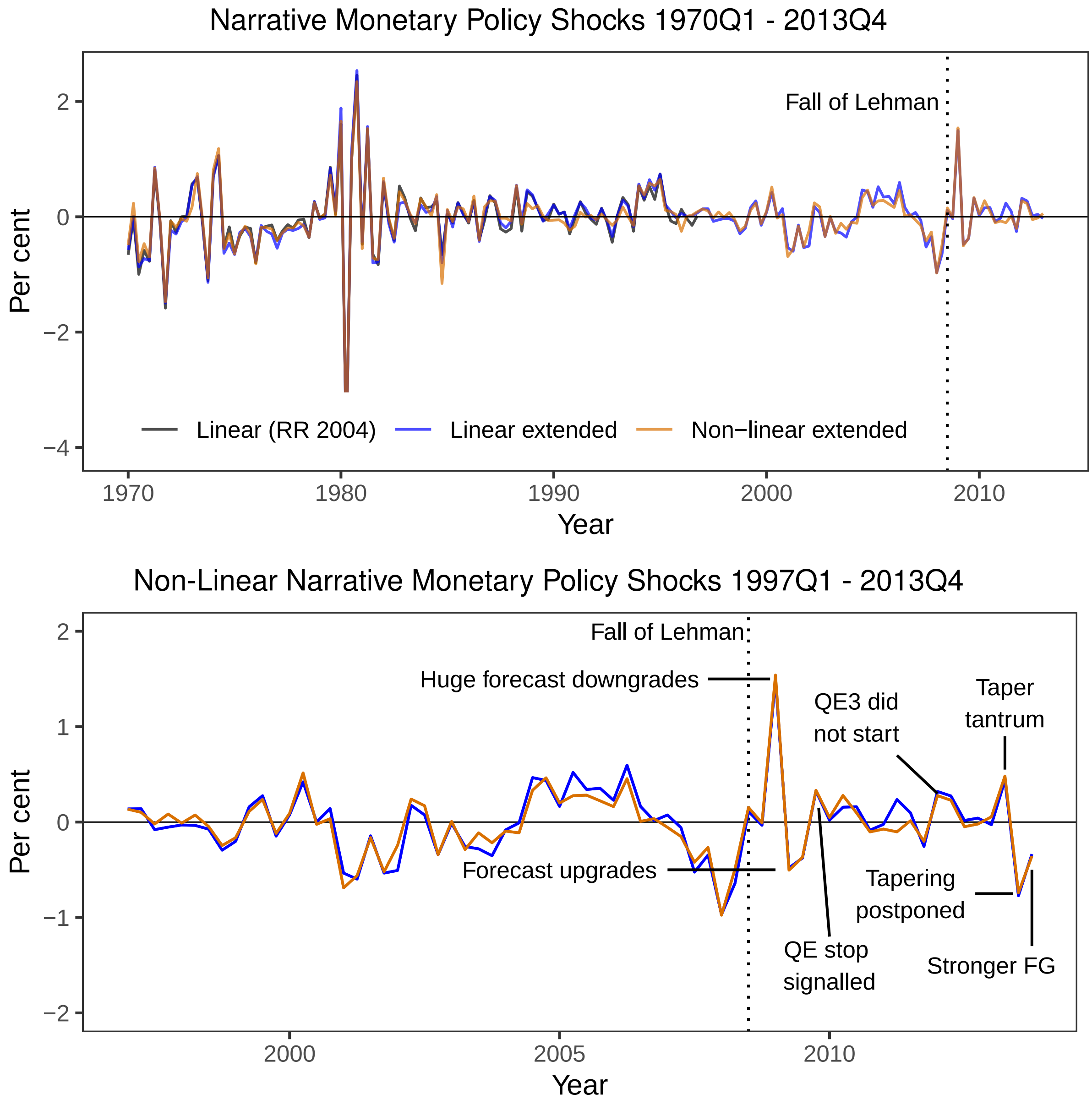

Figure 2. Narrative Monetary Policy Shocks. Top: Whole Sample, Bottom: Recent Sample.

Note: The narrative monetary policy shocks extend Romer and Romer (2004) up to 2013Q4. The top figure shows the RR original shocks (black line), the extended linear narrative shock (blue line), and the extended non-linear narrative shocks (orange line). The bottom figure zooms in to show how the narrative approach with shadow rates neatly captures unexpected movements in unconventional monetary policies since the global financial crisis.

In the literature, an alternative monetary policy shocks identification for post-crisis is to use high frequency data around monetary policy announcements (Gertler and Karadi, 2015; Nakamura and Steinsson, 2018). The high frequency identification literature often refers to the changes in futures contracts around key monetary events. They use a tight window around these events, in order to isolate monetary policy news from other types of shocks (Cesa-Bianchi et al., 2020). However, data for high-frequency identification only goes back to the 1990s, as these financial instruments were not actively traded before then, if at all. While this is sufficient for monthly estimation, this paper uses the SPF which is performed quarterly. Using the narrative monetary policy shock identification (instead of high frequency) allows me to use the full sample from the 1970s. Additionally, Ramey (2016) highlights that the high-frequency identification (HFI) shocks imply very similar effects of monetary policy (on output), despite the different samples and identification methods.

Figure 2 shows how the narrative approach with shadow rates neatly captures unexpected movements in unconventional policies since the financial crisis.[19] We observe a large positive shock in the first quarter of 2009. In March, the FOMC observed an increasing economic slack and this was reflected in a significant downgrade of economic forecasts – real GDP growth at two quarters ahead was downgraded to -0.5% instead of +1.8% – indicating that the FOMC realised that the U.S. economy was in a deep recession. This led to their decision of announcing an additionally large QE. However, the QE was not strong enough to overcome the contractionary effect of the Delphic forward guidance (Campbell et al., 2012). By 2009Q2, the FOMC saw a modest improvement in the economic outlook since the March meeting, reflected in their forecasts upgrades, which partly reflected some easing of financial market conditions. However, economic activity was likely to remain weak for a time, thus the magnitude was smaller than the preceding quarter. By the end of 2009, in light of ongoing improvements in the financial markets, the FOMC signalled that the special liquidity facilities will expire in 2010Q1. Nonetheless, they communicated that they were prepared to modify plans if necessary to support financial stability and economic growth, which helps explain the small positive (contractionary) shock.

Another example of how the narrative approach captures monetary policy shocks is shown in the first half of 2012, where there is a sequence of positive shocks. In these periods the FOMC did not start QE3 (the third round of QE) as the market had hoped multiple times. The relatively dovish statement was largely expected by markets. Combined, this is reflected in the modest contractionary shocks in the periods.

Moreover, in June 2013, there were discussions of 'tapering' QE purchases, contingent on a continuation of good economic data.[20] These discussions surprised financial markets, and in effect, producing what would be widely known as the "taper tantrum". However, in September 2013, the FOMC held off from scaling back asset purchases – again, surprising market participants, but in the opposite direction. Correspondingly, these two unexpected announcements generated a positive shock

[19] My notation focuses on the events after the financial crisis. Economic events in the periods between 1997 and 2007 have been discussed in the aforementioned papers.

[20] Specifically, the FOMC plan to reduce the pace of purchases of Treasuries from $85 billion per month to $65 billion by the second half of 2013, and further possibility of completely stopping asset purchases in 2014.

(contractionary) in June 2013, and a negative shock (expansionary) in September 2013 in the generated RR shocks in Figure 2.

2.4 Local Projections

Local projections method has recently been applied to study the state-dependent effects of monetary policy as it can be easily adapted for estimating a state-dependent model. I use Jordà's (2005) local projections method to estimate the impulse response to estimate the response of output and inflation for each horizon h.[21] The regressions for each horizon h that allows for state-dependence is as follows:

$$\begin{aligned} x_{t+h} = & F(z_{t-1})[\alpha_{A,h} + \psi_{A,h}(L)X_{t-1} + \beta_{A,h} mps_t] \\ & + (1 - F(z_{t-1}))[\alpha_{B,h} + \psi_{B,h}(L)X_{t-1} + \beta_{B,h} mps_t] + \tau + \tau^2 + \varepsilon_{t+h} \end{aligned} \tag{3}$$

where x is the variable of interest – log real GDP and log GDP deflator (in levels). The state variable $F(z_{t-1})$ equals 1 when the economy is in regime A (high disagreement periods) and 0 when in regime B (low disagreement periods). X is a vector of control variables and $\psi_h(L)$ is a polynomial in the lags operator specifically assigned to the variable of interest x. In particular, for GDP, the control variables are lag polynomial of order 2 in disagreement and lag polynomial of order 4 in GDP and FFR in the high and low disagreement periods. For GDP deflator, the control variables are lag polynomial of order 1 in disagreement, GDP deflator and FFR for the two disagreement regimes. τ and τ^2 is a linear and a quadratic trend. ε_{t+h} is the residual term. The coefficient β_h gives the response of x at time $t + h$ to the narratively-identified monetary policy shocks mps at time t. The impulse responses are constructed as sequences of the β_h's estimated in a series of single regressions for each horizon. The interactions with the indicator variable allows all coefficients to vary according to the state of the economy (high vs low disagreement periods).[22] The set of coefficients $\beta_{A,h}$ and $\beta_{B,h}$ are used to construct the impulse responses for each regime A and B, respectively. I use the Newey-West standard error correction to address the potential autocorrelation in the residuals (Newey and West, 1994).

[21] In Section 4, I investigate the research question using an endogenous threshold VAR with recursive monetary shocks and find that the main results hold. Hence, considering that local projections is easily adaptable for state-dependent estimation, and that narratively identified monetary shocks exogeneity, this method is preferable than the alternative, e.g. threshold VAR.

[22] In the main analysis, this is the threshold high and low disagreement, while Appendix A.5 uses a smooth transition state using a logistic transformation as in Tenreyro and Thwaites (2016).

3 Results

The main empirical results document how during heightened real GDP nowcast disagreement, monetary policy has *smaller* effect over inflation, yet *stronger* influence over output. During high disagreement periods, output responds fairly quickly to the narrative monetary policy shocks. Whereas in a low disagreement regime, output is statistically insignificant from zero for more than three years. This result arises from the higher stickiness of prices in the high disagreement periods.

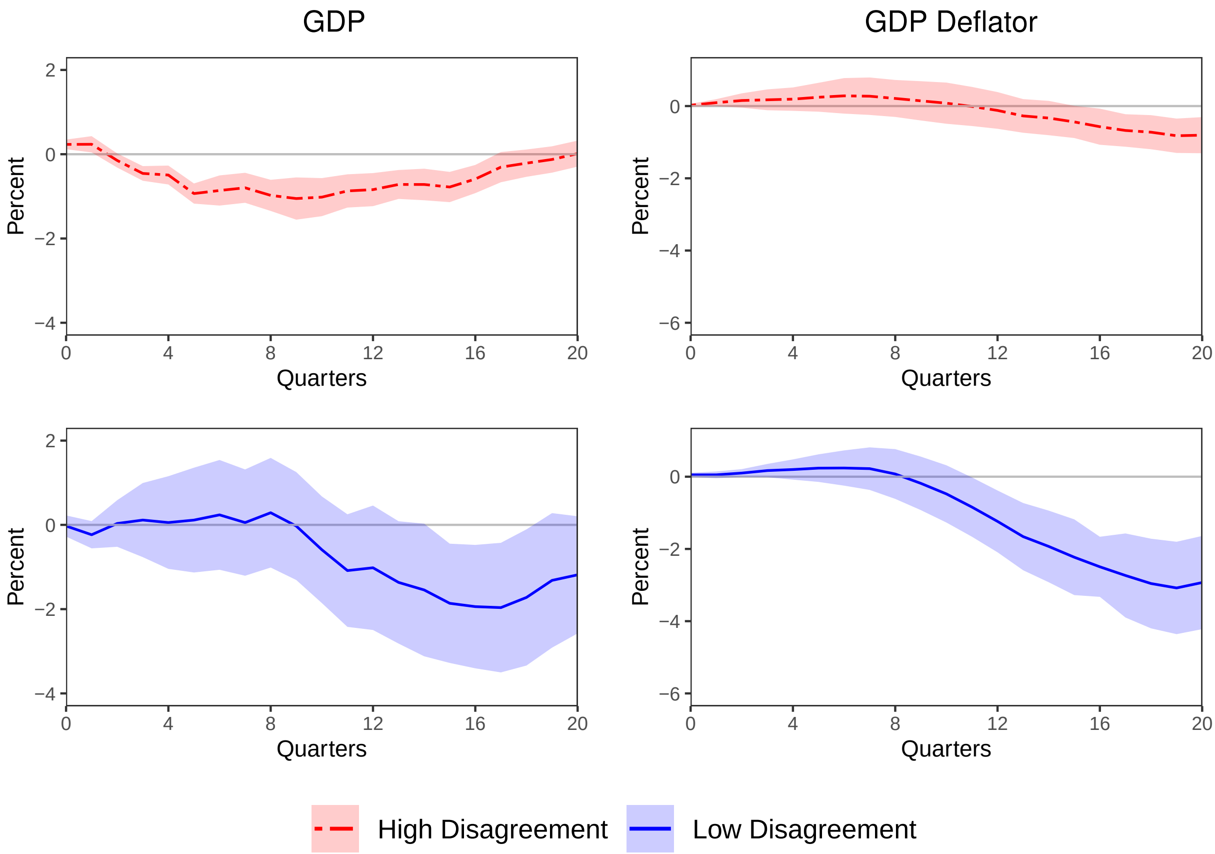

Figure 3. Local Projection Impulse Responses to a Narrative Monetary Policy Shock

Note: The first and second column shows the response of real output and prices to a 1% non-linear narrative monetary policy shock. The first and second rows show the responses under high (red-dash lines) and low (blue-solid lines) disagreement periods, respectively. The shaded area is the 68% confidence interval. The sample period is from 1970Q1 to 2013Q4.

Figure 3 shows the impulse responses of real GDP and GDP deflator to a 1% narrative monetary policy shock.[23] The upper row shows the impact of monetary policy shocks in a high disagreement regime – a period defined as having disagreement higher than the median real GDP nowcast disagreement from 1970Q1 to 2013Q4. The

[23]The linear results in Appendix A.4 illustrate a combination of results from the literature. The non-linear responses show how it masks the different behaviour in the two information frictions states.

lower row shows the IRFs in a low disagreement regime. The shaded area around the impulse responses is the 68% confidence bands.

In high disagreement periods (red lines), output initially rises but quickly declines in response to a monetary policy shock before slowly recovering towards the end of the estimation period. At its trough, output falls by 1 percent in 1.5 years. In contrast, during low disagreement (blue lines) output is statistically insignificant from zero for more than twelve quarters. This result arises from the higher stickiness of prices during the high disagreement periods, as apparent from the magnitude of the impulse responses towards the end of the profile.

In terms of prices, although in both regimes prices decline slowly and persistently, prices respond faster in low disagreement periods, by approximately three quarters. By the end of the estimation period, the divergence between the two regimes is approximately 2pp. During high disagreement periods, prices fall to 0.8 percent. Meanwhile, the impact during low disagreement periods is three times stronger as it falls to 2.9 percent.

Notice that using the narrative monetary policy shocks method, we do not observe a price-puzzle, which often appears when using a recursive estimation method. In Figure 3, the response of prices is statistically insignificantly different from zero for a few quarters. This is one of the advantages of the Romer-Romer narrative monetary policy shocks.

The magnitude of the impulse responses is comparable to Tenreyro and Thwaites (2016) who also use local projections to estimate the state-dependent effects of monetary policy shocks in recessions and expansions. For output, they find a maximum fall of 1 percent to a 1 percentage point rise in the Federal Funds Rate, and for prices, their finding is larger (about 8 percent in an expansion against 4 percent in a recession). It is important to note, however, that the state of recession and expansion is not directly linked to high and low disagreement as the correlation between these states of the world is weak.

4 Digging Deeper

So far I have focused on disagreement about output nowcast. However, measuring disagreement and choosing a relevant series is not straightforward in the literature. How do different measures of disagreement affect the results? Additionally, this section explores to what extent the results depend on sample periods and econometric methods.

4.1 Alternative Measures of Disagreement

A priori, it is indiscernible which forecast horizons to use when measuring disagreement. On one hand, survey variables with a longer forecast horizon may contain more noise-to-signal that could affect the identification of the state of the economy. Figure 9 plots the one-year ahead real GDP forecast disagreement. In comparison to the real GDP nowcast disagreement, the level of disagreement is almost four times higher, and there is a more frequent movement between high and low disagreement in 1980s to 1990s. Bok et al. (2018) find forecasts of real GDP in the SPF are most helpful to understand where the economy is now (nowcast) and Eusepi and Preston (2018) show SPF long-term expectations drift overtime. Moreover, it is complex to forecast GDP in all horizons as it is released with a delay and subject to major revision that can cause much uncertainty surrounding GDP nowcasts and forecasts (for instance, see Bank of England's Monetary Policy Report fan chart). On the other hand, Andrade et al. (2016) find professional forecaster in the Blue Chip survey disagree more about output growth (and inflation) in the near-term than in the long-term. Disagreement about inflation expectations is well researched (among many, Mankiw et al. (2004) and Falck et al. (2021)), but Dovern et al. (2012) find different underlying sources of output and inflation disagreement. Thus, how disagreement about different macroeconomics variable affects the transmission of monetary policy remains an empirical question.

To what extent do different disagreement measures affect state-dependent monetary policy transmission? I re-estimate the response of inflation and output using one-year ahead real GDP forecast (instead of nowcast), as well as disagreement about inflation (instead of output). Figure 4 shows the IRFs to a 1% narrative monetary policy shock using different disagreement measures as the state-dependent variable.[24] The solid lines are the responses using real GDP nowcast as the disagreement measure and the shaded area shows the confidence interval. The dashed lines are the responses using real GDP one-year ahead forecast disagreement and the dotted line using inflation expectation one-year ahead disagreement.[25]

The responses are broadly in line with the main results – monetary policy has smaller effects on inflation but a stronger impact on output under heightened disagreement. Under heightened disagreement, the responses of prices and output under disagreement about output forecast and inflation expectations are well within the confidence bands of the baseline results. However, in low information frictions peri-

[24] For comparison purposes, instead of using non-linear narrative monetary policy shocks, I use the *linear* shocks. The correlation between linear and non-linear narrative monetary policy shocks is 0.99.

[25] I have also re-estimated the results using two-quarter ahead inflation expectations disagreement following the baseline exercise of Falck et al. (2021). I find the response of output in the two regimes to be similar.

ods, the effect of monetary policy shocks under different measures of disagreement is slightly varied. Using real GDP forecast disagreement, the overall dynamic of output and inflation is similar to the main results but it is more subdued. This could be due to less predictability in the forecasts beyond the near-term. Yet when professional forecasters disagree about inflation expectations, the response of inflation is weaker (with more price-puzzle) than when disagreement is about output. The response of output is even more strikingly different from the main results as monetary policy shocks produce a contractionary effect in the low disagreement regime, that is, output does not respond differently in the two regimes. This is in contrast to the main finding where output response is stronger in heightened disagreement, but is in line with the baseline results of Falck et al. (2021).

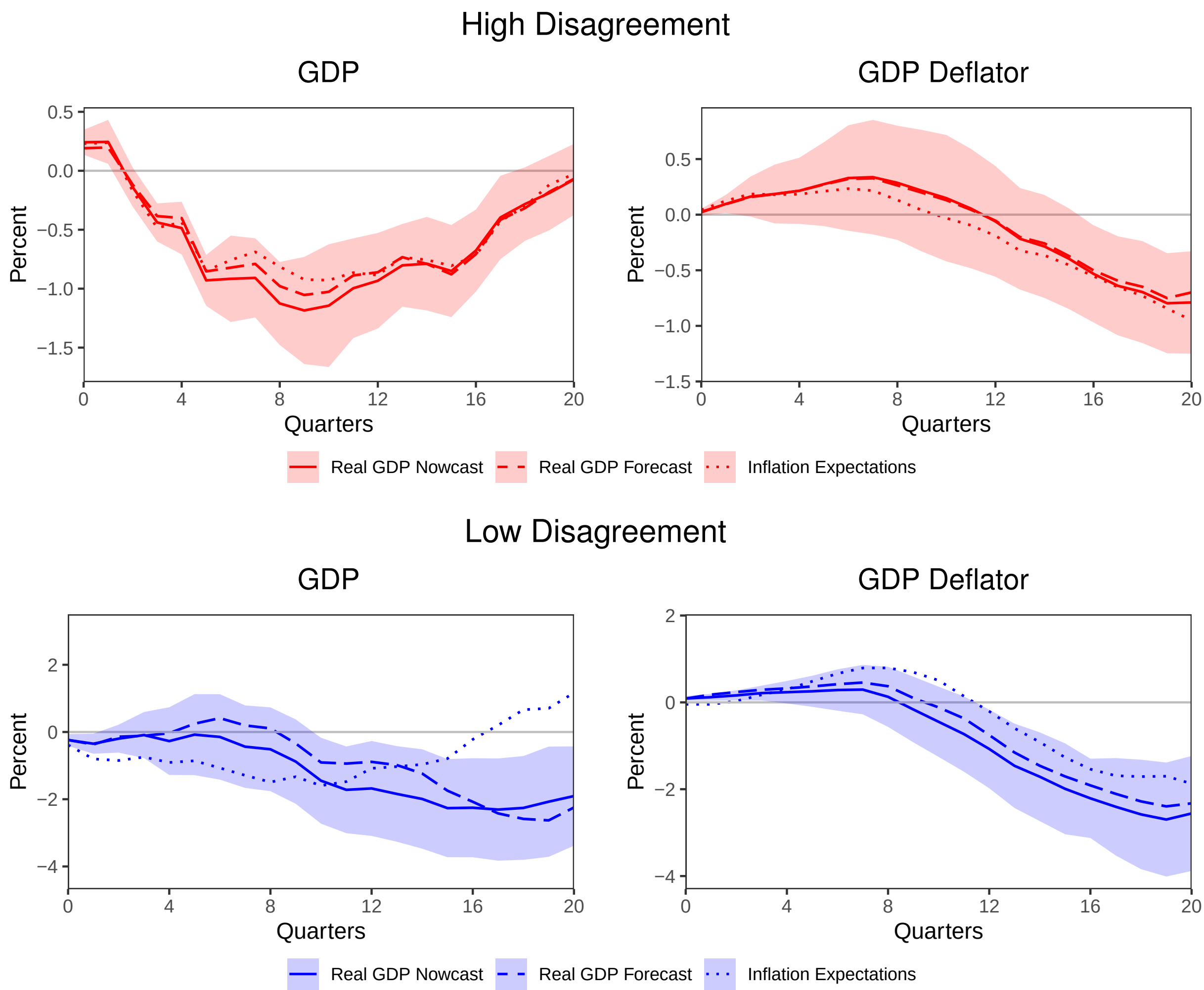

Figure 4. Local Projection Impulse Responses to a 1% Linear Narrative Monetary Policy Shock with Alternative Measures of Disagreement as Threshold

Note: The first (second) column shows the response of real output (prices) to a 1% linear narrative monetary policy shock. The first (second) row show the responses during high (low) disagreement periods. The lines correspond to using real GDP nowcast (solid), real GDP 1y ahead forecast (dashed) and 1y ahead inflation expectations (dotted) SPF. The shaded area is the 68% confidence interval when using real GDP nowcast as the threshold variable. The sample period is from 1970Q1 to 2013Q4.

4.2 Alternative Sample Periods

The beginning of the sample period (1970s-1980s) sees many episodes of high disagreement. To investigate whether the main result was solely driven by the early years of the survey, I re-estimate the impulse responses using data from 1983Q1 to 2013Q4. Starting the sample from 1983 is consistent with explanation in Section 2. While the baseline disagreement series in Figure 1 shows that the moderation of disagreement starts in the early 1990s, a Wald test points towards a much earlier structural break in the sample (1980Q2). Additionally, Ramey (2016) and Coibion (2012) point out the narrative monetary policy shock of Romer and Romer (2004) could be very sensitive to the inclusion of the period of non-borrowed reserves targeting in 1979-82. Starting the sample in 1983 also excludes periods of Great Inflation, as Stock and Watson (2002) find that Great Moderation starts in 1983. Evidently, Figure 8 shows that the median disagreement across the sample period is smaller.

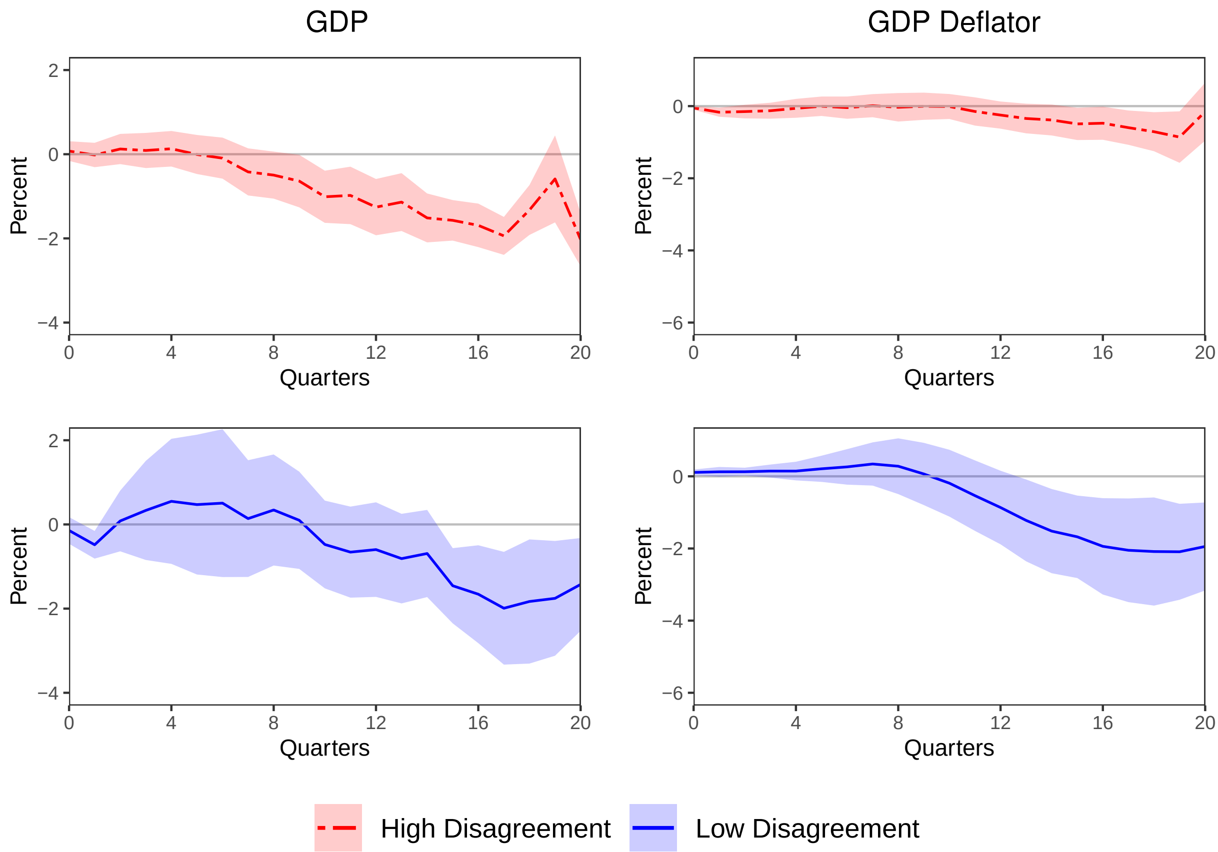

Figure 5. Local Projection Impulse Responses to a 1% Narrative Monetary Policy Shock Post-Great Inflation

Note: The first and second column shows the response of real output and prices to a 1% narrative monetary policy shock. The first and second rows show the responses under high (red lines) and low (blue lines) disagreement periods, respectively. The shaded area is the 68% confidence interval. The sample period is from 1983Q1 to 2013Q4.

Figure 5 shows that the main result was not driven by the episodes of 'very high' disagreement in the 1970s to early 1980s. Using the shorter sample, the results still show that monetary policy has a weaker effect on inflation but stronger impact over output during heightened information rigidities. In comparison to the main result, there is only a small quantitative difference in the response of output in a high disagreement period. Output remains insignificant (from zero) for two years – that is, it takes longer for output to display a contractionary effect.

4.3 Threshold VAR

This subsection investigates whether the main results can be replicated using another leading methodology – endogenous threshold VAR – to capture the potentially different effects of monetary policy shocks in high and low disagreement regimes. The advantage of this method is that parameters are allowed to differ across (disagreement) regimes, and the transition between the regimes is governed by the evolution of a single *endogenous* variable of the VAR crossing a threshold (the 'threshold variable').[26]

The general idea of the empirical threshold VAR methodology is to pick an endogenous 'threshold variable' that contains information about the different regimes (Tsay, 1998) – in this case, high and low disagreement.[27] Therefore, this makes it possible that regime switches may occur after the shock to each variable. Because of this, the magnitude (and even the sign) of the impulse response may be affected by: (1) the state of the system at the time of the shock, (2) the sign of the shock, and (3) the magnitude of the shock.

The estimation of the threshold uses conditional maximum likelihood (Galvão, 2006). If the threshold is known, it is possible to simply split the sample (above and below the threshold variable) and estimate the parameters with OLS, as well as the variance-covariance matrix Σ of the residuals U_t in each of the two regimes. Thus, a numerical optimiser iterates across the threshold values, to find the optimal threshold θ^*.

$$\theta^* = \min_{\theta} \left[\frac{T_1}{2} \log |\widehat{\Sigma}^{(1)}(\theta)| + \frac{T_2}{2} \log |\widehat{\Sigma}^{(2)}(\theta)| \right] \tag{4}$$

where $|\widehat{\Sigma}^{(i)}(\theta)|$ is the determinant of the covariance matrix of the residuals U_t in regimes $i = 1, 2$ (low and high disagreement regimes). The delay parameter is set to

[26] There are other non-linear methodologies, including smooth-transition VARs, interacted VARs, and Markov-switching approaches. The choice of appropriate non-linear methodology depends on the specific research question.

[27] The method allows for endogenous regime switching which implies that the response of economic variables can depend on the sign and magnitude of the structural shock, unlike linear VARs. This flexible methodology allows us to examine the potentially different properties of the transmission of contractionary/expansionary monetary policy shocks.

1, hence the regimes change with a lag of one period, after crossing the threshold.

The first term on the right hand side of the equation in the threshold VAR model is analogous to a linear VAR. The non-linearity of the model comes from introducing different regimes on the second term of the right hand side.

$$Y_t = \left[c_1 + \sum_{j=1}^{p} \gamma_1(L) Y_{t-j}\right] + \left[c_2 + \sum_{j=1}^{p} \gamma_2(L) Y_{t-j}\right] I(y^*_{t-d} > \theta^*) + U_t \tag{5}$$

where Y_t is a vector of endogenous (stationary) variables. Real GDP, GDP deflator and commodity price index is transformed with log first-differences. I is an indicator function that takes the value of 1 when the threshold variable is higher than the *estimated* threshold parameter θ^*, and 0 otherwise, with time lag d set to 1. U_t are reduced-form disturbances. $\gamma_1(L)$ and $\gamma_2(L)$ are lag polynomial matrices with order p. The lag order selection by Akaike information criteria marginally chose 4 lags in the linear VAR, and to maintain consistency, I estimate the threshold VAR with the same number of lags. As this is a non-linear model, I use the generalised impulse responses (GIRFs) approach of Tsay (1998). The full algorithm, including the computation of bootstrap confidence intervals, is described in Appendix C of Caggiano et al. (2015).

The exercise here uses recursive identification for the threshold VAR analysis.[28] The specific recursive identification – real GDP, GDP deflator, commodity price index, FFR and disagreement – reflects some assumptions about the links in the economy. The ordering of the first four variables associated with the Cholesky decomposition of the covariance matrix of U_t is widely used, such as in Bernanke and Gertler (1995).[29] Ordering disagreement last implies that it reacts contemporaneously to all other variables. The results are robust to other orderings.

Figure 6 shows the generalised impulse response functions (GIRFs) from the threshold VAR. It corroborates the main results of heterogeneity in the effects of monetary policy shocks in the two disagreement regimes.[30] The GIRFs correspond to a 1 standard deviation positive shock to FFR. The shaded area around the impulse responses is the 68% confidence bands. The estimation period is set to 20 quarters.

[28] I also run threshold VAR with narrative monetary shocks in Appendix A.6. The main results broadly hold.

[29] The Cholesky decomposition in this paper assumes lower triangular matrix, such that monetary policy shocks do not affect real GDP, GDP deflator and commodity price index within the same quarter.

[30] Notice that these shocks are monetary policy shocks rather than monetary policy changes. The monetary policy shocks is relative to what the Taylor rule implies should happen, and the Taylor rule is implicit in the (threshold) VAR in Eq (5). Thus, in times of weak growth, it is perfectly feasible to have a positive monetary policy shock (that is, monetary policy could have loosened but not as much as the implicit Taylor rule suggests).

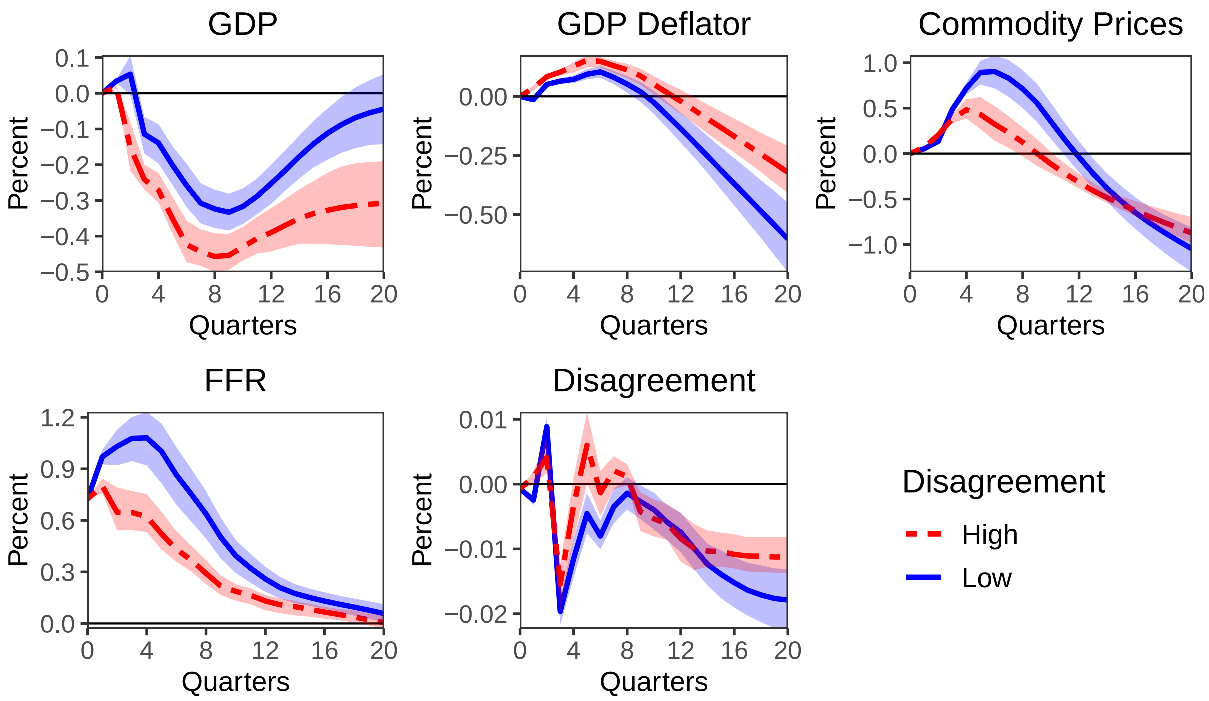

Figure 6. Threshold VAR Generalised Impulse Responses to a Monetary Policy Shock

Note: The shock corresponds to a positive one standard deviation change in the FFR. The GIRFs are generated with 68% bootstrapped confidence intervals using threshold VAR. The threshold is estimated using SPF disagreement of the nowcasts of **real** GDP. Red dashed-line indicate high disagreement period and blue solid-line low disagreement period. Sample period is between 1970Q1 and 2018Q4.

The peak impact of the contractionary monetary policy shocks reduces GDP by approximately 0.3% in the low disagreement regime. Whereas in a high disagreement regime, an equivalent sized shock reduces GDP by 0.45% at the trough – a sizeable difference of around half. Furthermore, the real effects of monetary policy are much more persistent under high disagreement, in addition to falling faster on impact. Correspondingly, the impact of monetary policy under low disagreement is *stronger*. At the end of the GIRFs horizon, the impact on prices is -1.2%, almost twice lower than the effect of -0.7% under high disagreement. This highlights how, in the presence of heightened disagreement, the trade-off between output and inflation worsens, as output falls faster after a positive monetary policy shock.

In comparison to these results, the responses of output and prices using the state-dependent local projections method with narrative monetary policy shocks seem to capture stronger effects. This is not a surprise because Romer and Romer (2004) find that using the narrative identification method produces a much larger effect of monetary policy than using recursive method, such as in Christiano et al. (1999).[31]

[31] Another possible explanation for the difference in the results is in the econometrics methodology (Ramey, 2016). The local projection procedure and the VAR procedure which has an analogy with direct forecasting (forecast future values of a variable using a horizon specific regression) vs it-

In addition to the transmission mechanism to output and inflation, the set up of the endogenous threshold VAR allows for the disagreement measure to respond to a monetary policy shock beyond the response of inflation and output. There are two observations. One, the response of FFR is higher for longer in the low disagreement regime. A potential explanation for this is, in a high disagreement regime, output falls significantly more and thus the endogenous monetary policy component is forced to relax monetary policy. On the other hand, in a low disagreement regime, the central bank may need to keep monetary policy tighter for longer to lower inflation. This suggests that, at least empirically, the inflation expectations channel does not operate by as much as the fall in inflation created by the drag on output gap. Two, there is a varied response of disagreement in the two information frictions regimes. In more detail, disagreement that started in a high disagreement period, increase slightly but fall significantly in the short run. Then, beyond the first year, it recovers before persistently falling below zero beyond the two years horizon. The responses of disagreement in the two disagreement regimes are not significantly different for up to a year, but the disagreement in the low disagreement regime is persistently negative after that.

5 Interpreting Results According to Rational Inattention

To illustrate the mechanisms that generate the empirical results, I design a tractable rational inattention model. The first part of this section presents a stylised price-setting model with rational inattention, with closed-form solutions that will allow for comparative statics. The model shows how disagreement endogenously evolves to changes in the information processing of firms and various uncertainties relevant for pricing decisions, and how that relates to how monetary shocks affect optimal prices. Then, I use the model to provide an explanation of the heterogeneous responses of inflation and output to a narrative monetary policy shock under time-varying disagreement. Moreover, a novel insight from the model is that it dissects the relationship between disagreement and uncertainty – two fundamentally different concepts, and gives further insight to the disagreement measure.

erated forecasting (forecast on a one-period ahead estimated model). The local projection method is analogous to the direct forecasting, whereas the standard VAR method is analogous to the iterated forecasting method. In understanding propagation of structural shocks, an often asked question is how to choose between SVAR and LP estimators of impulse responses. From the impulse response estimation perspective, Plagborg-Møller and Wolf (2020) prove that LPs and VARs estimate the same impulse responses. They also show that the two are not conceptually different methods.

5.1 Stylised Rational Inattention Model

In this model, price-setters in the firms face an unobserved aggregate demand y_t, composed of a normally-distributed demand shock b_t, and a 'monetary policy' component $c \cdot r_t$. The demand shock has a variance σ_b^2, which I refer to as fundamental demand uncertainty.[32] For tractability, without loss of generality, the demand shock is assumed to be mean-zero. The monetary policy component is fully known: price-setters observe the policy rate r_t and the interest-elasticity of demand $c > 0$.

$$y_t = b_t - c \cdot r_t, \qquad \text{where} \quad b_t \sim N(0, \sigma_b^2) \tag{6}$$

In this simple model, I assume demand is insensitive to prices, leading to a flat demand curve. The full-information optimal price p_{it}^* purely depends on the marginal costs, which is increasing with respect to demand y_t, and decreasing to an unobserved, stochastic firm-specific productivity term a_{it} where i represents a firm.

$$p_{it}^* = \varphi y_t - a_{it}, \qquad \text{where} \quad a_{it} \sim N(0, \sigma_a^2) \tag{7}$$

This simple structure can be micro-founded by a profit-maximising firm with decreasing returns to scale (thus marginal costs are increasing in output) that is common with rational inattention models, or a firm that faces labour market rigidities (thus needs to pay higher wages to produce more output).

To help set optimal prices, firms receive the signals $s_{it} = \{s_{it}^y, s_{it}^a\}$ on key variables:

$$s_{it}^y = y_t + \varepsilon_{it}^y, \qquad \varepsilon_{it}^y \sim N(0, \nu_{y,t}^2) \tag{8}$$

$$s_{it}^a = a_{it} + \varepsilon_{it}^a, \qquad \varepsilon_{it}^a \sim N(0, \nu_{a,t}^2) \tag{9}$$

The firms choose the variance of the noise on the two signals, but this decision is subject to an information constraint:[33]

$$I\left(p_{it}^*; s_{it}\right) = H\left(p_{it}^*\right) - H\left(p_{it}^* \mid s_{it}\right) \le K \tag{10}$$

where the firms are limited to how much entropy $H(\cdot)$ they could reduce on the two state variables b_t and a_{it} after observing the signal s_{it}. Given that the signals are uncorrelated and Gaussian, this will have the functional form of Eq (11):[34]

[32] The simplifying assumption that the shock is white noise, making it possible to get analytical solutions, as the optimal information decision is independent across time periods. I abstract away from dynamics, as I am interested in the intratemporal attention allocation.

[33] Notice that the firms do not receive signals about other firms idiosyncratic shocks, or public signals, and thus do not create higher-order signal extraction problems.

[34] I leave the derivation details in Appendix B.

$$\underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_y^2}{\nu_{y,t}^2}+1\right)}_{K_{it}^y}+\underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_{a_i}^2}{\nu_{a,t}^2}+1\right)}_{K_{it}^a}\leq K \tag{11}$$

where K_{it}^y and K_{it}^a are the entropy reduction to the uncertainty on the two unobserved state variables. Hereafter, I refer to K_{it}^y and K_{it}^a as the 'attention' firm i allocates to monitoring y_t and a_{it}, which will be chosen optimally.[35]

Rearranging Eq (11), the attention allocations imply the following perceived volatility of the tracking noises:

$$\nu_{y,t}^2=\frac{1}{2^{2K_{it}^y}-1}\sigma_y^2 \tag{12}$$

$$\nu_{a,t}^2=\frac{1}{2^{2K_{it}^a}-1}\sigma_{ai}^2 \tag{13}$$

In other words, the more attention paid to each variable, the associated variance of the noise on the signals would be lower. As the signals are i.i.d., and the only source of information on y_t is s_{it}^y, any dispersion in the expectations of y_t across firms i is captured by $\nu_{y,t}^2$. Thus, $\nu_{y,t}^2$ is a sufficient summary statistic of demand nowcast disagreement.

5.1.1 Optimal Pricing and Attention Allocation

Each firm i minimises the expected profit losses due to mispricing by setting prices given its information choice, subject to the maximum information gain constraint:

$$\min_{\{K_{it}^y,K_{it}^a\}\in\mathcal{R}^+} E\left[(p_{it}-p_{it}^*)^2|s_{it}\right] \qquad \text{subject to } K_{it}^y+K_{it}^a\leq K \tag{14}$$

Minimising the quadratic loss around the full-information optimal price subject to information constraints is equivalent to profit-maximisation. The quadratic loss function is symmetric, so it is trivial to show that the optimal price is the firms' best guess of what the true optimal price is given the signal it receives:

$$p_{it}=E\left[p_{it}^*\mid s_{it}\right]=\varphi E\left[y_t\mid s_{it}^y\right]-E\left[a_{it}\mid s_{it}^a\right] \tag{15}$$

[35] In the Zhang (2017) model, K is pinned down by ensuring the marginal benefit of information equates to a fixed marginal cost of information, as the firms 'purchase' information with a linear cost in K. This model has a small, but important, departure by assuming maximum information gain constraint K is exogenous to the firm. This makes it more tractable to see the impact of changes in uncertainty of different variables, as well as changes in the information capacity, on attention allocation and price-setting.

The model is solved by a backward two-step procedure. Firstly, the optimal price is solved for a given attention allocation $\{K_{it}^y, K_{it}^a\}$. Secondly, I use the result from the first step to substitute for the profit loss (from the optimal profit) in the firm's objective as a function of the information choice. The attention allocation decision can then be solved by optimising the objective.

The optimal price setting decision for a given attention allocation can be inferred from standard Bayesian updating and the pricing rule Eq (15). Rearranging it, we get Eq (17) which can be attained using noise volatilities from Eq (12) and Eq (13):

$$p_{it} = \varphi \frac{\sigma_y^2}{\sigma_y^2 + \nu_{y,t}^2} s_{it}^y - \frac{\sigma_a^2}{\sigma_a^2 + \nu_{a,t}^2} s_{it}^a \tag{16}$$

$$= \varphi \left(1 - 2^{-2K_{it}^y}\right) s_{it}^y - \left(1 - 2^{-2K_{it}^a}\right) s_{it}^a \tag{17}$$

This optimal pricing behaviour is substituted into the expected profit loss due to mispricing, noting the independence of fundamental and noise shocks, results in:

$$E\left[(p_{it} - p_{it}^*)^2 \mid s_{it}\right] = \varphi^2 2^{-2K_{it}^y} \sigma_y^2 + 2^{-2K_{it}^a} \sigma_a^2 \tag{18}$$

$$= \varphi^2 2^{-2K_{it}^y} \sigma_b^2 + 2^{-2K_{it}^a} \sigma_a^2 \tag{19}$$

where the last equality Eq (19) results from the prior variances $\sigma_y^2 = \sigma_b^2$, as the monetary policy component of demand $c \cdot r_t$ is observable. Substituting the maximum information gain constraint, it is trivial to show the expected profit loss is strictly convex for any finite and strictly positive combination of $\{\sigma_b^2, \sigma_a^2\}$. Thus, there exists a unique interior solution for the optimal attention allocation:[36]

$$K_{it}^{y*} = \frac{1}{2} \log_2 \left(\frac{\varphi \sigma_b}{\sigma_a}\right) + \frac{1}{2} K \tag{20}$$

$$K_{it}^{a*} = \frac{1}{2} \log_2 \left(\frac{\sigma_a}{\varphi \sigma_b}\right) + \frac{1}{2} K \tag{21}$$

The optimal attention allocation results are very intuitive: the attention paid to demand is increasing with the total attention available K and the uncertainty surrounding demand σ_b (as higher demand uncertainty increases the benefits to monitoring demand conditions y_t), while decreasing in productivity uncertainty σ_a. The last result suggests that an increase in productivity uncertainty would make firms reallocate attention away from monitoring demand conditions.

[36]See Appendix B for details of the derivation.

5.1.2 Comparative Statics: Disagreement

In this subsection, I examine how disagreement of demand conditions $\nu^2_{y,t}$ responds to changes in: (1) total attention available K, (2) productivity uncertainty σ^2_a, and (3) demand uncertainty σ^2_b. In the next subsection, I delve into the price reaction to monetary policy shocks in response to changes in the aforementioned parameters.

Firstly, for demand disagreement, I revisit Eq (12). From this equation, it is clear that disagreement is a function of (exogenous) fundamental uncertainty, but also related to the endogenous decision of attention allocation:

$$\nu^2_{y,t} = \frac{1}{2^{2K^y_{it}} - 1}\sigma^2_y$$

Substituting in the optimal attention allocation and differentiating it with respect to K, σ^2_a and σ^2_b results in:

$$\frac{d\nu^2_{y,t}}{dK} = -\sigma^2_b \ln(2) 2^{2K^y_{it}} \left(\frac{1}{2^{2K^y_{it}} - 1} \right)^2 < 0 \tag{22}$$

$$\frac{d\nu^2_{y,t}}{d\sigma^2_a} = \frac{1}{2}\frac{\sigma^2_b}{\sigma^2_a} 2^{2K^y_{it}} \left(\frac{1}{2^{2K^y_{it}} - 1} \right)^2 > 0 \tag{23}$$

$$\frac{d\nu^2_{y,t}}{d\sigma^2_b} = \frac{-2 + 2^{2K^y_{it}}}{2(2^{2K^y_{it}} - 1)^2} \gtrless 0 \tag{24}$$

The first two derivatives, Eq (22) and (23), are simple and fairly intuitive: changes in total information processing available to firms K and productivity uncertainty σ^2_a affect demand disagreement only through the endogenous response of attention K^y_{it}. A lowering of the total information processing capacity of firms leads firms to pay less attention to aggregate demand (as well as productivity). This leads to a poorer quality of information and thus increased disagreement across firms. Similarly, an increase of fundamental idiosyncratic productivity uncertainty lead firms to reallocate attention away from monitoring aggregate demand conditions, which also increase demand disagreement.

The more interesting case is what happens when fundamental demand uncertainty σ^2_b rises. The sign of the derivative in Eq (24) is ambiguous: it is positive when $K^y_{it} > \frac{1}{2}$ and negative when $K^y_{it} < \frac{1}{2}$. In other words, when attention on aggregate demand is relatively high, fundamental demand uncertainty *positively* co-moves with demand disagreement, but when attention is relatively low, uncertainty and disagreement *negatively* co-move. This is because there are two opposing forces: a direct effect of an increase in fundamental uncertainty, and an indirect effect from the endogenous re-allocation of attention towards monitoring demand. When attention is relatively low, the re-allocation of attention towards aggregate demand conditions could be strong enough that it overturns the direct effect (as the marginal benefits of re-allocating attention towards demand is high).

5.1.3 Comparative Statics: Price Setting

This subsection returns to the key research question: how do prices respond to monetary shocks under different conditions? By combining Eq (15) and $s^y_{it} = y_t + \varepsilon^y_{it} = b_t - cr_t + \varepsilon^y_{it}$, we arrive at:

$$\frac{dp_{it}}{dr_t} = \frac{dp_{it}}{ds^y_{it}} \cdot \frac{ds^y_{it}}{dr_t} = \left(1 - 2^{-2K^y_{it}}\right) \cdot (-c)\varphi < 0 \tag{25}$$

$$= -\varphi c \left(1 - \frac{\sigma_a}{\sigma_b \varphi} 2^{-K}\right) \tag{26}$$

where we derive the second line by substituting in K^{y*}_{it} from Eq (20). Intuitively, firms set lower prices as demand falls (as full-information optimal prices also fall). However, the extent that this occurs depends on the level of attention on aggregate demand conditions.

Taking the second-order comparative statics of Eq (26) with respect to the same parameters in the previous subsection:

$$\frac{d^2 p_{it}}{dr_t dK} = -\ln(2)\frac{\sigma_a}{\varphi\sigma_b} 2^{-K}\varphi c < 0 \tag{27}$$

$$\frac{d^2 p_{it}}{dr_t \sigma_a} = \frac{1}{\varphi\sigma_b} 2^{-K}\varphi c > 0 \tag{28}$$

$$\frac{d^2 p_{it}}{dr_t d\sigma_b} = -\frac{\sigma_a}{\varphi}\frac{1}{\sigma_b^2} 2^{-K}\varphi c < 0 \tag{29}$$

These results are also fairly intuitive: prices are less responsive to monetary shocks when firms pay less attention to aggregate demand condition (thus raises disagreement). This could be generated by: (1) a reduction in total information processing capacity, (2) an *increase* in productivity uncertainty, or (3) a *decrease* in aggregate demand uncertainty.

5.2 Insights from the Rational Inattention Model

5.2.1 Explaining the Main Findings

The rational inattention model offers three explanations for the main empirical findings. All explanations have a common theme that to produce a more sluggish response of prices to a monetary shock, attention paid by price-setters to aggregate conditions must be lower. Thus, firms react less to monetary shocks, making prices more 'sticky'. A standard New Keynesian model with stickier prices would predict that output would respond more to a monetary policy shock.

First, the information processing capacity of firms K could be lower, leading firms to reduce attention to aggregate conditions (and others). This could be caused by a variety of reasons – for example, the exit of firms over the business cycle breaks down existing supplier-customer relationships that facilitate information flows across the supply chain. This would also reduce the quality of the information that the firm processes, leading to higher disagreement, which is consistent with the empirical finding that prices would then be more sluggish.

Second, higher uncertainties in state variables *other than* aggregate conditions (in the model, idiosyncratic productivity σ_a^2 was one example), lead firms to re-allocate attention away from aggregate conditions. This has the same effect in increasing disagreement and stickier prices. This result also holds in larger general equilibrium models. Maćkowiak and Wiederholt (2009) show that to explain the sluggish response of prices to aggregate monetary shocks, it must be that idiosyncratic productivity matters a lot more for firm profits than demand uncertainty ($\sigma_a^2 \gg \sigma_b^2$), implying that firms pay little attention to aggregate conditions. While my model is not quantitative, the result in Maćkowiak and Wiederholt (2009) at least points to the plausibility of negative co-movement between uncertainty and disagreement.

Third, a *decrease* in aggregate demand uncertainty σ_b^2 could potentially make prices more sticky. A rationally inattentive firm would respond to this by reducing attention allocated to monitoring aggregate conditions. The model shows that in some parameter regions, the endogenous response of attention allocation has the potential to increase disagreement by reducing the information quality used to monitor aggregate conditions. These regions typically occur when the overall variance of aggregate conditions is low compared to idiosyncratic shocks, thus the marginal benefits of paying attention are high. This is exactly the parameter space that Maćkowiak and Wiederholt (2009) suggest are plausible to create the effect that prices respond sluggishly to monetary shocks.

5.2.2 Disagreement and Uncertainty

A novel insight from the rational inattention model is that it dissects the relationship between disagreement and uncertainty – two fundamentally different concepts.[37] The mechanisms of increased disagreement and uncertainty to the monetary transmission mechanism can be very different. An illustration where uncertainty and disagreement do not co-move together is the case where there is a reduction of information processing capability of agents (raising disagreement and weakens monetary policy trans-

[37] Empirically, Kozeniauskas et al. (2018) document there is only a low correlation between various measures of uncertainty and disagreement.

mission), even when the fundamental uncertainty on macroeconomic outcomes has *not* changed. Therefore, the effect of rising uncertainty on the responsiveness of prices is potentially non-monotonic, and the three different posited mechanisms in Section 5.1.2 could be more important at different times. These results bridge disagreement with the broader literature on the effect of uncertainty on monetary transmission, which typically finds that monetary policy has a weaker effect on prices and output during heightened uncertainty (Aastveit et al., 2017; Castelnuovo and Pellegrino, 2018).[38]

5.2.3 Explaining Disagreement

The rational inattention model also provides an insight for the additional results in Section 4. Using nowcast output disagreement in the main exercise, I find the response of output is stronger during high disagreement periods. However, when agents disagree about inflation, output do not respond differently in the two regimes. This could be due to the different underlying sources of disagreement in SPF inflation and output (Dovern et al., 2012; Falck et al., 2021). Turning to the tractable rational inattention model, price-setters optimise the profit-maximising price with subject to the signals from aggregate demand condition and their firm-specific productivity. The total signal of price-setting decision would be captured in total attention capacity K_{it}. Therefore, using inflation expectation disagreement would be a proxy for total attention, rather than disagreement about demand nowcast. While it may capture how total attention capacity varies overtime, it would be difficult to identify the source of the movement of information rigidities that would be useful for monetary policymaking. Additionally, Maćkowiak and Wiederholt (2009) show that firms respond more to an idiosyncratic stochastic volatility than to monetary shocks – thus, a movement in K_{it} may largely depict attention to firm-specific signals.

Furthermore, the tractable rational inattention model provides some intuition on the non-monotonic response of disagreement of demand condition to a monetary shock that we observe in the threshold VAR GIRFs. For instance, in a period of higher inflation or macroeconomic uncertainty, people are already paying a lot of attention to the current conditions, such that following a monetary policy shock, they may not benefit from allocating more attention to it. In other words, when attention on aggregate demand condition is already relatively high, paying even more attention may result in a lower marginal benefit. Hence in lower disagreement periods, disagreement initially falls because a small increment in attention may lower their current disagree-

[38] These results sit in between empirical findings from the literature on the state-dependent effects of monetary policy, where there appears little agreement across the literature. For example, Caggiano et al. (2014) and Tenreyro and Thwaites (2016) find that monetary shocks have less impact on (both) output and prices in *recessions*, while others such as Peersman and Smets (2001) and Lo and Piger (2005) find the opposite.

ment. Then, as agents realise that the marginal benefit of paying additional attention is low, their disagreement would rise to a higher level than in the heightened disagreement periods. However, at the end of the horizon, disagreement is persistently lower in the low information rigidities periods. A policy implication from this medium-run dynamic would be that a central bank would benefit from having a communication strategy that lowers disagreement.

6 Conclusion and Policy Implications

The main contribution of the paper is to empirically document the state-dependent effects of monetary policy during varying nowcast disagreement about real output. Using non-linear local projections and narrative monetary policy shocks, I show that in periods of heightened disagreement, monetary policy has smaller effects on inflation, but larger impact on output. The interpretation from a rational inattention model of the empirical result is that price-setters respond less in periods with higher information frictions, and thus prices become stickier. These stickier prices lead to smaller price adjustments, but also because of the higher nominal rigidities, it causes a flatter Philips curve, leading to larger output effects for the given monetary shock.

A novel insight from the rational inattention model is that endogenous optimal attention allocation could cause disagreement to change non-monotonically in response to fluctuations in aggregate uncertainty. A by-product of the model dissects the relationship between uncertainty and disagreement, and how they distinctly affect state-dependent monetary transmission under varying degrees of information frictions.

The key policy takeaway from these results is the role of central bank communication. The results show that during periods of low disagreement, contractionary monetary policy (that intends to reduce inflation, "disinflationary policy") is able to reduce inflation significantly with relatively little output loss. This raises the potentially important role of central bank in communicating aggregate conditions to economic agents, enabling them to internalise the disinflationary (contractionary) policy that effectively makes prices more flexible. Thus, the sacrifice ratio is lower and enables an inflation-targeting central bank to better achieve its objective. This mechanism complements the literature results in having a credible central bank moving inflation expectations down during a disinflationary policy episode, which further reduces the sacrifice ratio.

Similarly, if inflation is below target but output is at potential, it is also optimal for the central bank to communicate. The increase in price flexibility allows it to increase inflation to target more quickly while avoiding large and unsustainable positive output gaps (which are associated with undesirable effects, such as misallocation and credit booms). However, if a dual-mandate central bank objective is to raise economic growth rather than stabilising inflation, it is not necessarily optimal either to *not* communicate. In a world of low interest rates, forward guidance could be a potent tool for expansionary monetary policy. Naturally, communication is an integral part of forward guidance. Thus, improving communication during such an episode, and achieving the benefits of forward guidance may outweigh the cost of increased price flexibility in terms of a reduction of the real effects of monetary policy.

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A Robustness Checks

A.1 GDP vs GNP

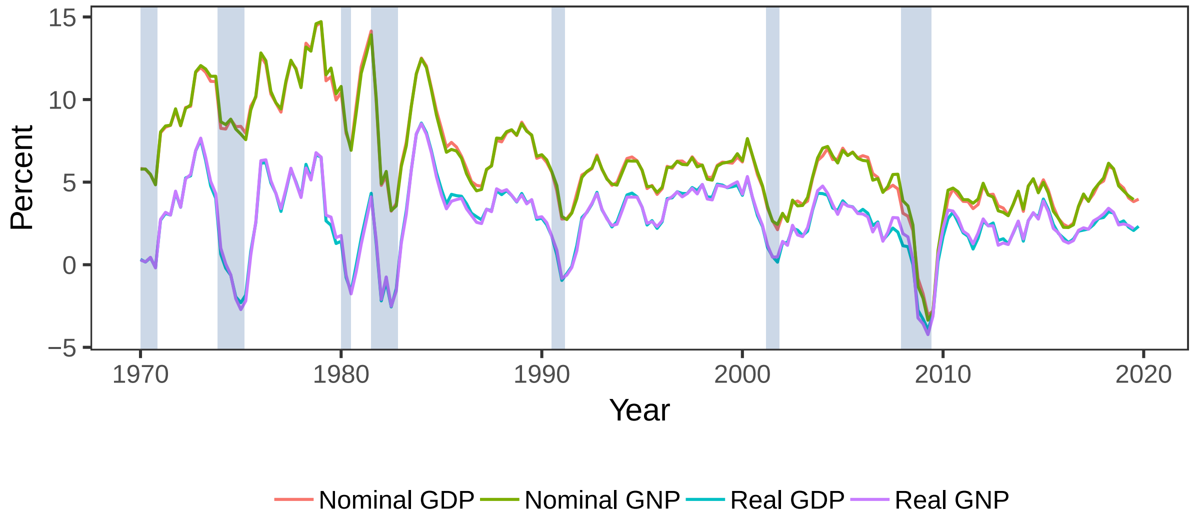

Figure 7. GNP and GDP 4-Quarter Growth

Note: The figure shows the percent change from a year ago of GDP and GNP. The red line depicts Nominal GDP and the blue line depicts the Real GDP. The green line depicts Nominal GNP and purple line the Real GNP. The sample period is 1970Q1-2018Q4.

A.2 Time Series of Disagreement with Alternative Sample Period

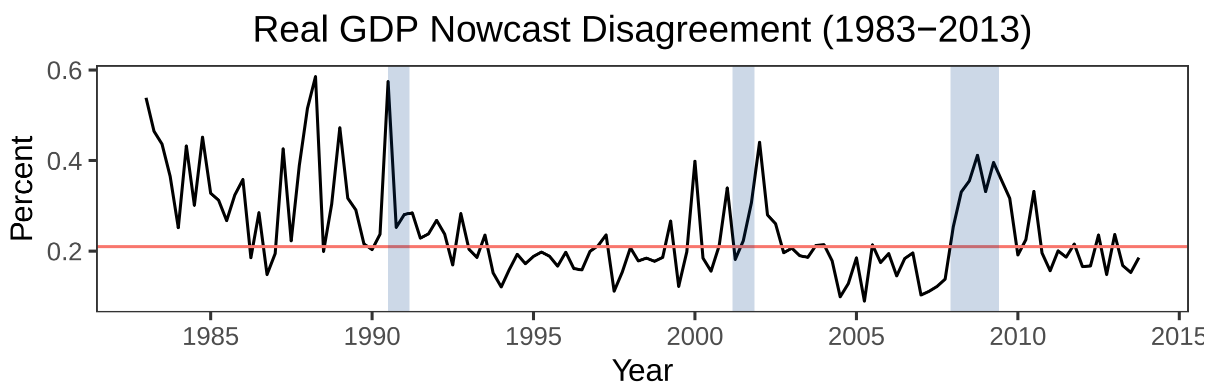

Figure 8. Time-Varying Real GDP Nowcast Disagreement (1983-2013)

Note: Time series of the real GDP nowcast disagreement index based on the dispersion (interquartile range) of the U.S. SPF between 1983Q1-2013Q4. The grey shaded areas indicate NBER-dated recessions. The solid red line is the median disagreement. The y-axis is the interquartile range as a percentage of the median.

A.3 Time Series of Disagreement with Alternative Variables

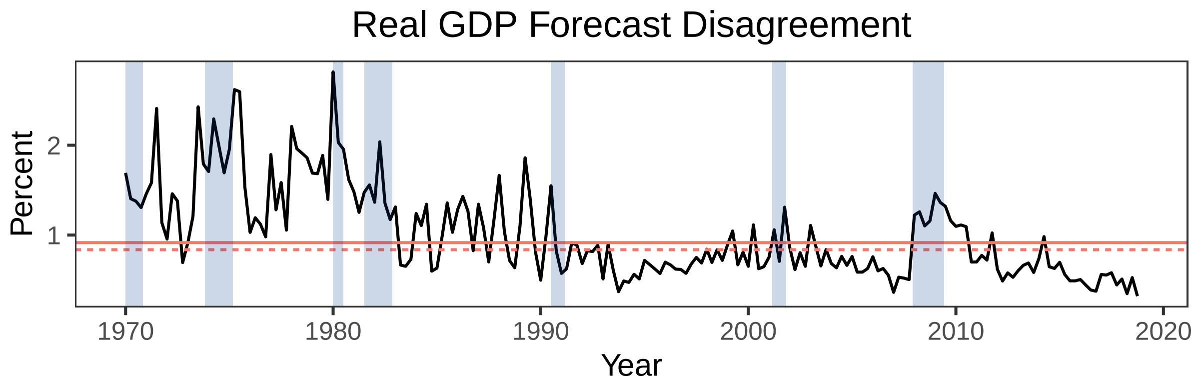

Figure 9. Time-Varying Disagreement: 1-Year Ahead Real GDP Forecasts

Note: Time series of the real GDP disagreement index based on the dispersion (interquartile range) of 1-year (4 quarters) ahead forecasts. The grey shaded areas indicate NBER-dated recessions. The solid red line indicates the median disagreement as the state-dependent variable for the local projections estimation, with the sample period of 1970Q1-2013Q4. The dotted red line indicates the estimated threshold in the threshold VAR with the sample period of 1970Q1-2018Q4.

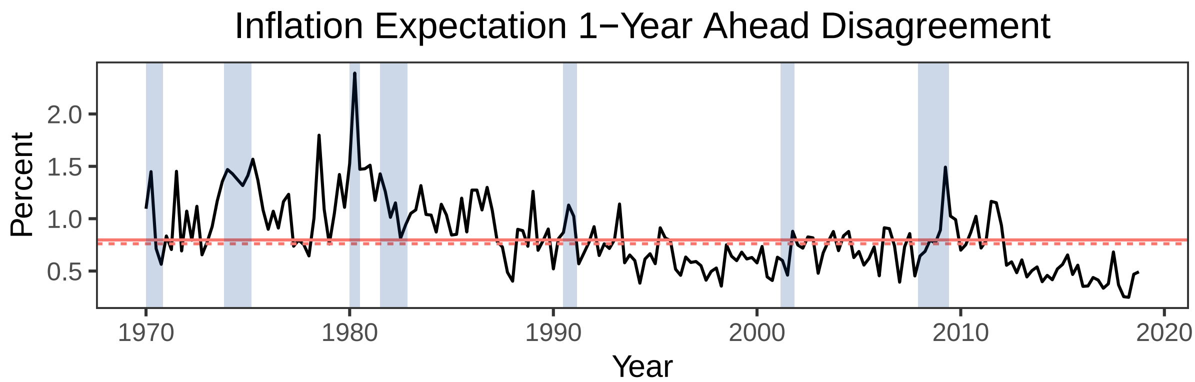

Figure 10. Time-Varying Disagreement: 1-Year Ahead Inflation Expectations Forecasts

Note: Time series of the inflation expectations disagreement index based on the dispersion (interquartile range) of 1-year (4 quarters) ahead forecasts. The grey shaded areas indicate NBER-dated recessions. The solid red line indicate the median disagreement as the state-dependent variable for the local projections estimation, with the sample period of 1970Q4-2013Q4. The dotted red line indicate the estimated threshold in the threshold VAR with the sample period of 1970Q4-2018Q4.

A.4 Linear responses

A useful starting point to examine how the responses of macroeconomic variables to monetary policy shocks may vary with information frictions is to use a linear method (which assumes that responses are invariant to the state of the economy). The non-linear responses show how the linear results masks the different behaviour in the two information frictions states.

Figure 11 shows the impulse response of output and prices to a 1% narrative monetary shock. Output declines briefly by the fifth quarter but quickly recovers. Whereas estimated impact on prices is virtually zero for the first 9 quarters after the shock. The effect becomes progressively more statistically significant as prices begin to fall substantially with the estimated impact is 1 percent after 2 years. Romer and Romer (2004) find similar dynamics using monthly VAR (1969-1996).

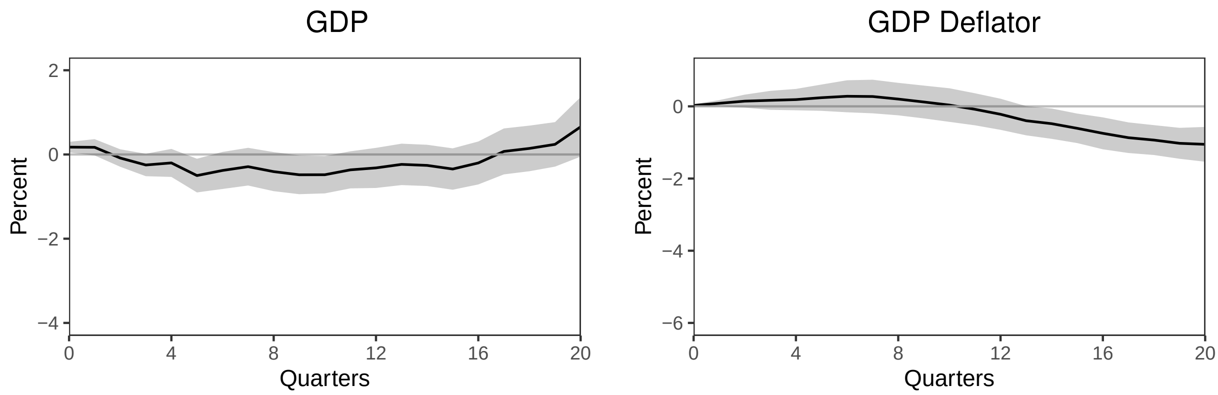

Figure 11. Local Projection Impulse Responses to a Narrative Monetary Shock

Note: The first and second column shows the response of real output and prices to a 1% linear narrative monetary shock. The shaded area is the 68% confidence interval. The sample period is from 1970Q1 to 2013Q4.

Similarly for the threshold VAR, the impulse responses in Figure 12 correspond to a 1 standard deviation positive shock to FFR. Note that the linear IRFs does not necessarily lie between the high and low disagreement because the GIRFs in Figure 6 allow for regime switching after a shock. In the linear VAR, the peak effect on real GDP is 0.5% after around 8 quarters (2 years), which is a typical horizon in the literature for output to respond to a contractionary monetary shock.

The commodity price index drops more quickly than GDP deflator as expected by Bernanke and Gertler (1995). The sluggish responses in real GDP and price level, as well as the persistent decline in GDP deflator is fairly consistent with the literature, for example Galí (2015) and Christiano et al. (1999). The GDP deflator depiction of a weak 'price-puzzle' – prices increase after an increase in FFR – is a common finding for monetary shocks identified with a recursive linear VAR.

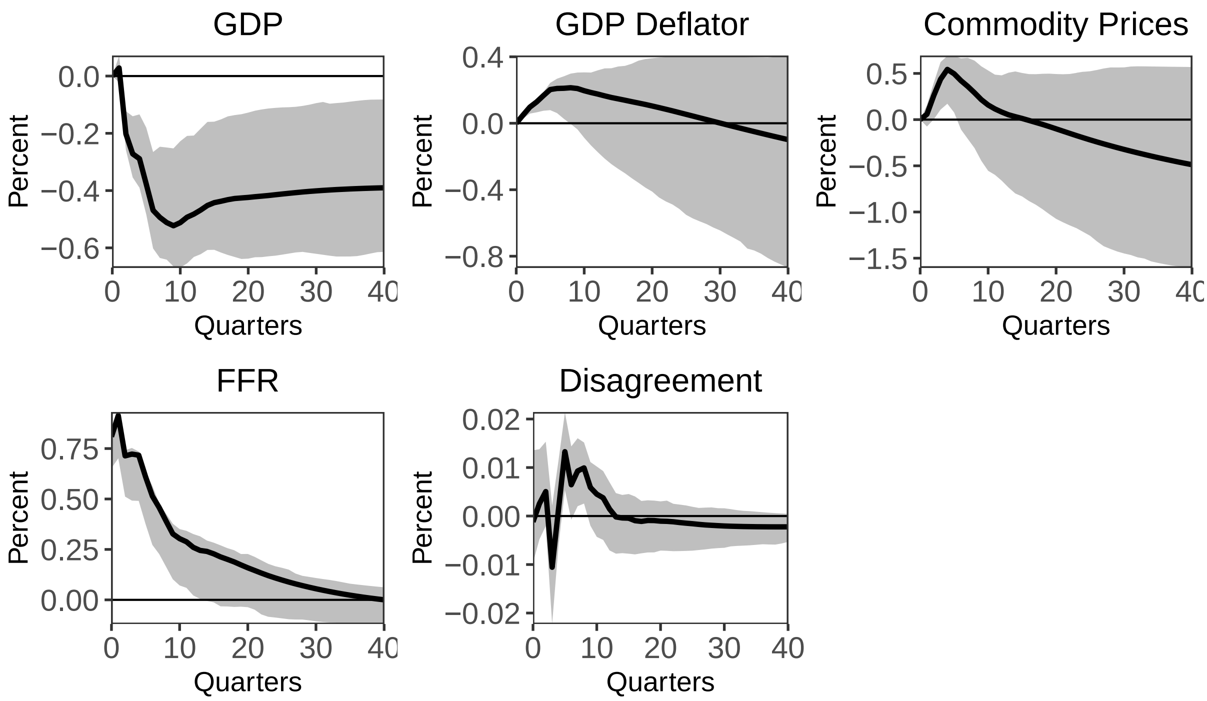

Figure 12. VAR Impulse Responses to a Monetary Policy Shock

Note: The shock corresponds to a positive one standard deviation change in the FFR. The IRFs are generated with 68% bootstrapped confidence intervals using (linear – without the distinction between high and low disagreement) Cholesky-identified structural VAR. Sample period is 1970Q1-2018Q4.

A.5 Smooth Transition Local Projections

One of the reasons to use a dummy variable in the local projections in the main section is to allow for the nature of the disagreement variables that may change at each period. But I show here that using the smooth transition local projections – which has been utilised in the literature to estimate the effects monetary (Tenreyro and Thwaites, 2016) and fiscal policy shocks in recession and expansion periods – the main results also hold. Figure 13 plots the narrative shock series, but for the non-linearly narratively identified monetary policy shocks, I use a smooth-transition method of regimes-switching.

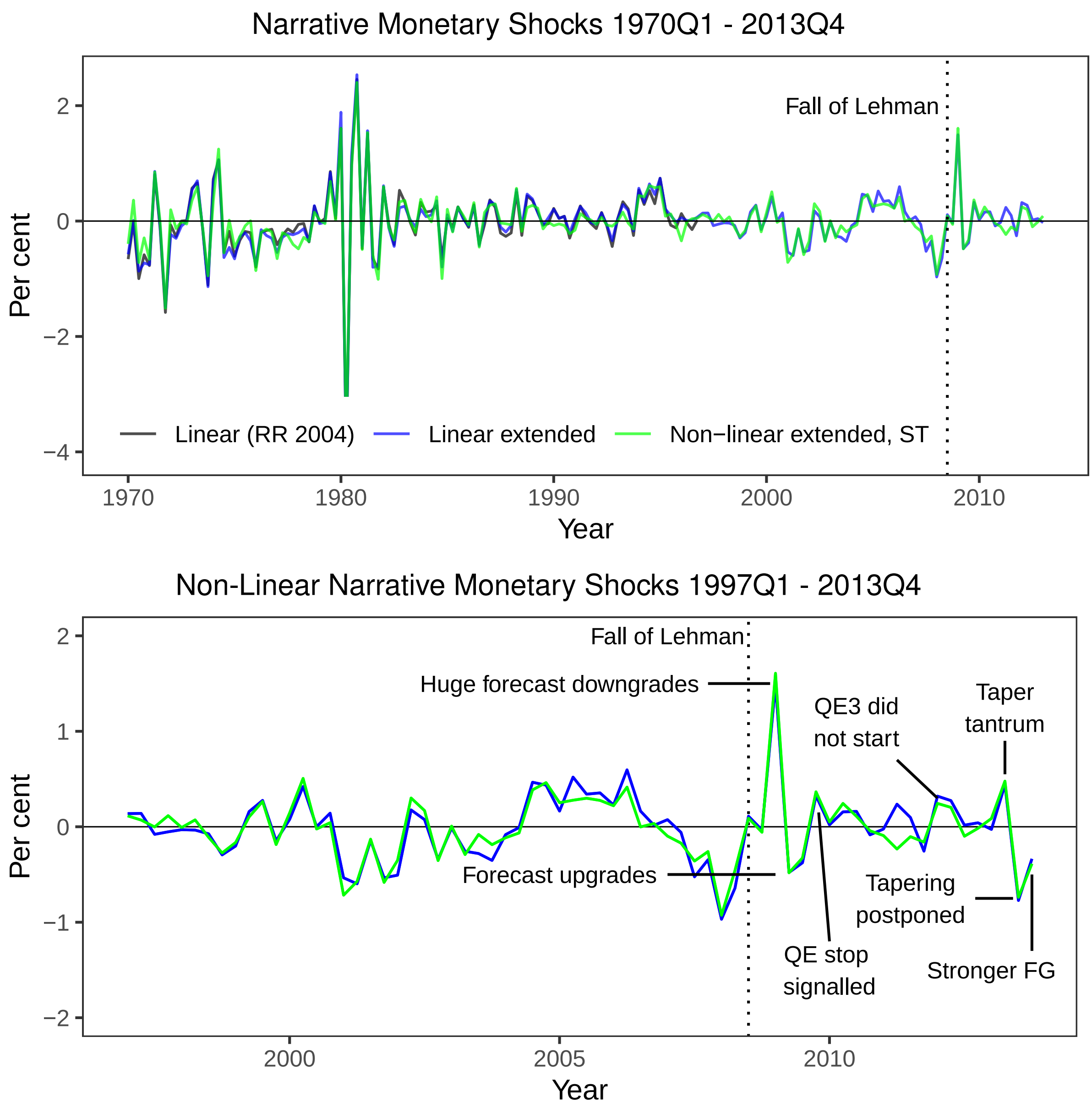

Figure 13. Smooth Transition Local Projections Narrative Monetary Shocks

I extend the narrative monetary shocks of Romer and Romer (2004) up to 2013Q4. The top figure shows the RR original shocks (black line), the extended linear narrative shock (blue line), and the extended non-linear narrative shocks (green line). I use smooth-transition to identify the disagreement regimes for the non-linearly narratively identified monetary shocks. The bottom figure shows how the narrative approach with shadow rates neatly captures unexpected movements in unconventional monetary policies since the financial crisis.

As in Section 2.4, I estimate a set of regressions for each horizon h as follows

$$x_{t+h} = F(z_{t-1})[\alpha_{A,h} + \psi_{A,h}(L)X_{t-1} + \beta_{A,h}\text{shock}_t] \\ + (1 - F(z_{t-1}))[\alpha_{B,h} + \psi_{B,h}(L)X_{t-1} + \beta_{B,h}\text{shock}_t] + \varepsilon_{t+h}$$

Instead here, $F(z_t)$ is a smooth increasing function of an indicator of the state of the economy z_t. Following Granger and Terasvirta (1993) and Tenreyro and Thwaites (2016), I employ the logistic function

$$F(z_t) = \frac{\exp\left(\theta\frac{(z_t-c)}{\sigma_z}\right)}{1+\exp\left(\theta\frac{(z_t-c)}{\sigma_z}\right)}$$

where c is a parameter that controls what proportion of the sample the economy spends in either state and σ_z is the standard deviation of the state variable $F(z_t)$. The parameter θ determines how violently the economy switches from high to low disagreement when z_t changes. Higher values of θ mean that $F(z_t)$ spends more time close to the $0, 1$ bounds of the process, moving the model closer to a discrete regime-switching setup. Smaller values of θ mean that more of the observations are taken to contain some information about behaviour in both high and low disagreement regimes. I calibrate the parameter value to $\theta = 3$, as in Tenreyro and Thwaites (2016), to give an intermediate degree of intensity to the regime switching.

Impulse Responses to a 1% Narrative Monetary Shock

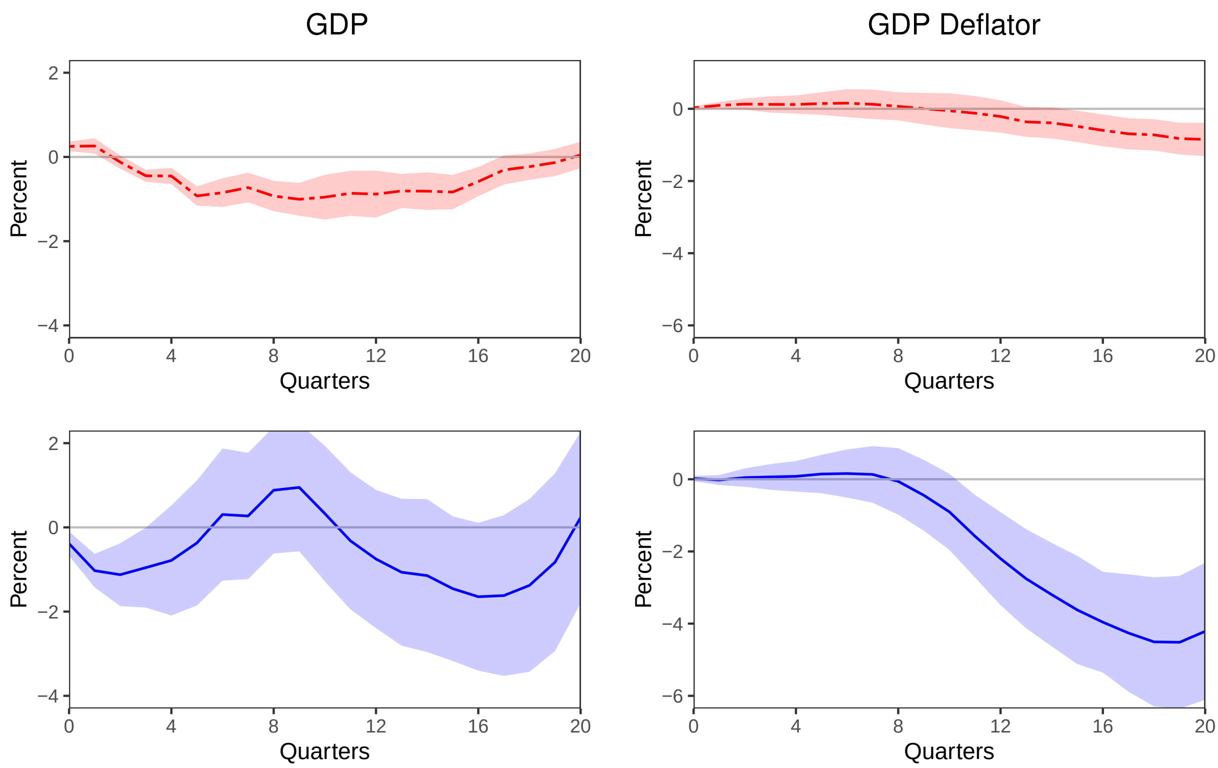

Figure 14. Smooth Transition Local Projections Impulse Responses

The first and second column shows the response of real output and prices to a 1% narrative monetary shock, respectively. The first and second rows show the responses under high and low disagreement periods, respectively. The shaded area is the 68% confidence interval. The sample period is 1970Q1-2018Q4.

A.6 Threshold VAR with Narrative Monetary Policy Shocks

This section combines the narrative monetary shock identification with the threshold VAR. Figure 15 shows the responses of output and prices to a positive one standard deviation shock to the narrative monetary shock. Consistent with the specification in the baseline exercise and Romer and Romer (2004), I use the narrative shocks instead of the FFR, but keeping the Cholesky ordering of the variables the same.[39]

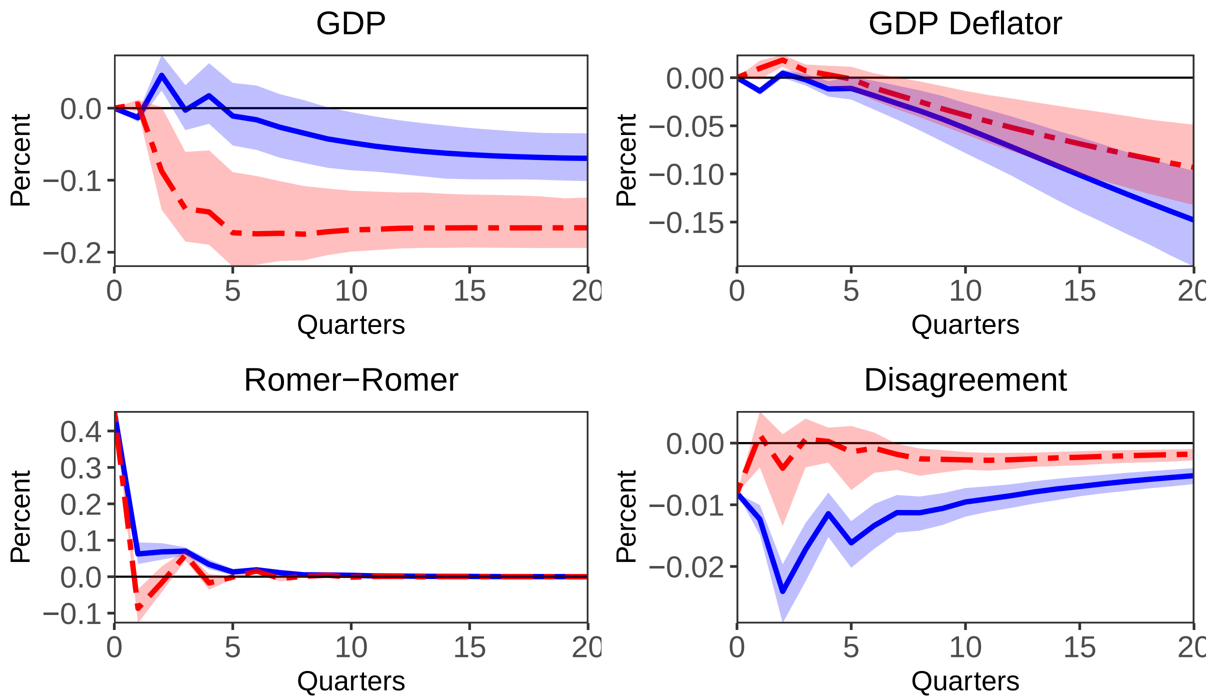

Figure 15. Threshold VAR Generalised Impulse Responses to a Narrative Monetary Shock

Note: The shock corresponds to a positive one standard deviation change in the narrative monetary shocks. The GIRFs are generated with 68% bootstrapped confidence intervals using threshold VAR. The threshold is estimated using SPF disagreement of the nowcasts of real GDP. Red-dash (blue-solid) lines indicate high (low) disagreement period. Sample period is 1970Q1-2013Q4.

Broadly, the responses of output and prices still demonstrates the heterogeneity in the effect of monetary policy shock across the high and low disagreement regimes. In high disagreement periods, prices respond weakly to monetary policy shock, but output responds strongly. In low disagreement, the opposite is true. Thus, the main result and mechanism as previously explained – that prices are more sticky in high disagreement periods due to higher information frictions, leading to larger real effects of monetary shocks – holds with narratively identified monetary shocks á la Romer and Romer (2004).

[39] I also removed commodity prices as a control because as Romer and Romer (2004) discussed, the narrative identification sufficiently avoids endogenous and anticipatory movements unlike the FFR, and therefore does not produce a large price-puzzle.

However, it is tricky to quantitatively compare the responses of using the narratively identified monetary shock to the Cholesky identified threshold VAR. One standard deviation shock in the narrative identified monetary shock (which is in changes-space) is not equal to the one standard deviation shock to FFR in levels. Additionally, because the GIRFs are inherently non-linear, we cannot simply scale the responses.

In Figure 15, at peak impact (around six-quarter horizon), the contractionary monetary policy reduces real GDP by approximately 0.2%, and similarly until the end of the horizon. During low disagreement periods, output eventually reduces by 0.07%, which is three times weaker compared to the response during high disagreement periods. Overall, the responses in the two regimes are significantly different from zero, and from each other. During high disagreement periods, output become immediately statistically significant from zero, while there is a lag during low disagreement periods. This is also observed in the generalised impulse response of output in Figure 6. In high disagreement, the peak response of output to the narrative monetary shocks is about half of the peak effect to the Cholesky identified monetary shocks.

The difference in magnitude is even more apparent in the response of prices to the two shocks. In low disagreement, the response of GDP deflator to the narrative monetary shock is -0.15%. More importantly, here we also observe the difference between the responses in high and low disagreement periods using the two shocks identification. Prices respond more strongly in low disagreement periods, and that it is significantly different from the response during high disagreement in the latter horizon. This suggests that both shocks identification strategies are able to pick up the heterogeneity in the responses of macroeconomic variables during different information frictions periods.

A.7 GIRF Bootstrap Algorithm

I follow the algorithm in Koop et al. (1996):

1. Pick a history and Ω_{t-1} contains the sequence of lagged data up to time $t-1$, which defines the history of the model at date t. Also, pick a structural shock of size δ.

2. Use Monte-Carlo integration to compute the *conditional* response for: variable y, shock size δ, history Ω_{t-1} and horizon $h = 0, 1, \ldots, H$

3. Then average out over each regime's set of random histories Ω^r, to get the *unconditional* responses for each regime

4. Subtract the **second** from **first** time path. The difference is the estimate of GIRF.

5. However, Step 4 is a noisy estimate. To eliminate the random variation in the GIRF, repeat steps **2 - 4** many times and take the mean of the resulting impulse responses as the central tendency. I also take the empirical quantiles from these draws to compute the confidence intervals.

B Rational Inattention Model Details

Optimal price setting decision:

$$p_{it} = E\left[p_{it}^* \mid s_{it}, I_{i,t-1}\right] = \varphi E\left[y_t \mid s_{it}^y, I_{i,t-1}\right] - E\left[a_{it} \mid s_{it}^a, I_{i,t-1}\right] \tag{30}$$

Information constraint:

$$I\left(p_{it}^*; s_{it} \mid I_{i,t-1}\right) = H\left(p_{it}^* \mid I_{i,t-1}\right) - H\left(p_{it}^* \mid s_{it}, I_{i,t-1}\right) \leq K \tag{31}$$

Note that for Gaussian distributed random variable X, the unconditional and conditional entropy is:

$$H(X) = \frac{1}{2}\log_2\left[2\pi e Var(X)\right] \tag{32}$$

$$H(X \mid I) = \frac{1}{2}\log_2\left[2\pi e Var(X \mid I)\right] \tag{33}$$

So:

$$\underbrace{H\left(y_t \mid I_{i,t-1}\right) - H\left(y_t \mid s_{it}^y, I_{i,t-1}\right)}_{K_{it}^y} + \underbrace{H\left(a_{it} \mid I_{i,t-1}\right) - H\left(a_{it} \mid s_{it}^a, I_{i,t-1}\right)}_{K_{it}^a} \leq K \tag{34}$$

Taking the profit maximising price and signals (where the noises of the signals follow unit-variance Gaussian processes and independent of one another), the information constraint becomes:

$$\underbrace{\frac{1}{2}\log_2\left[2\pi e Var\left(y_t \mid I_{i,t-1}\right)\right] - \frac{1}{2}\log_2\left[2\pi e Var\left(y_t \mid s_{it}^y, I_{i,t-1}\right)\right]}_{K_{it}^y}$$
$$+ \underbrace{\frac{1}{2}\log_2\left[2\pi e Var\left(a_{it} \mid I_{i,t-1}\right)\right] - \frac{1}{2}\log_2\left[2\pi e Var\left(a_{it} \mid s_{it}^a, I_{i,t-1}\right)\right]}_{K_{it}^a} \leq K$$

$$\underbrace{\frac{1}{2}\log_2\left[2\pi e \sigma_y^2\right] - \frac{1}{2}\log_2\left[2\pi e \frac{\nu_y^2}{\nu_y^2 + \sigma_y^2}\sigma_y^2\right]}_{K_{it}^y}$$
$$+ \underbrace{\frac{1}{2}\log_2\left[2\pi e \sigma_{ai}^2\right] - \frac{1}{2}\log_2\left[2\pi e \frac{\nu_{ai}^2}{\nu_{ai}^2 + \sigma_{ai}^2}\sigma_{ai}^2\right]}_{K_{it}^a} \leq K$$

$$\underbrace{-\frac{1}{2}\log_2\left[\frac{\nu_y^2}{\nu_y^2+\sigma_y^2}\right]}_{K_{it}^y}\underbrace{-\frac{1}{2}\log_2\left[\frac{\sigma_{ai}^2}{\sigma_{ai}^2+\sigma_{ai}^2}\right]}_{K_{it}^a}\leq K \tag{35}$$

$$\underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_y^2}{\nu_y^2}+1\right)}_{K_{it}^y}+\underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_{ai}^2}{\nu_{ai}^2}+1\right)}_{K_{it}^a}\leq K \tag{36}$$

Based on the previous equation, an attention allocation implies the following perceived volatility of the tracking noises

$$\nu_y^2=\frac{1}{2^{2K_{it}^y}-1}\sigma_y^2 \tag{37}$$

$$\nu_{ai}^2=\frac{1}{2^{2K_{it}^a}-1}\sigma_{ai}^2 \tag{38}$$

B.1 Optimal Pricing Rule and Attention allocation

For a given attention choice, Kalman filtering equation, pricing rule, and the noise volatility above, the optimal price setting decision is

$$\begin{aligned}
p_{it} &= E\left[p_{it}^* \mid s_{it}, I_{i,t-1}\right] \\
&= \varphi E\left[y_t \mid s_{yit}, I_{i,t-1}\right]-E\left[a_{it} \mid s_{ait}, I_{i,t-1}\right] \\
p_{it} &= \varphi\left(1-2^{-2K_{it}^y}\right)s_{it}^y-\left(1-2^{-2K_{it}^a}\right)s_{it}^a
\end{aligned} \tag{39}$$

Conditional profit loss due to mispricing becomes:

$$E\left[\left(p_{it}-p_{it}^*\right)^2 \mid I_{i,t-1}\right] \tag{40}$$

$$=E\left[\varphi\left(1-2^{-2K_{it}^y}\right)s_{it}^y-\left(1-2^{-2K_{it}^a}\right)s_{it}^a-\left(\varphi y_t-a_{it}\right)\right]^2 \tag{41}$$

$$=E\left[\varphi\left(-2^{-2K_{it}^y}y_t+\left(1-2^{-2K_{it}^y}\right)y_{it}\right)-\left(-2^{-2K_{it}^a}a_{its}+\left(1-2^{-2K_{it}^a}\right)a_{it}\right)\right]^2 \tag{42}$$

$$=E\left[\varphi^2\left(2^{-4K_{it}^y}y_t^2+\left(1-2^{-2K_{it}^y}\right)^2 y2_{it}\right)+\left(2^{-4K_{it}^a}a_{it}^2+\left(1-2^{-2K_{it}^a}\right)^2 a2_{it}\right)\right] \tag{43}$$

taking expectations and substituing ν_y^2 and σ_a^2

$$E\left[\left(p_{it}-p_{it}^*\right)^2 \mid I_{i,t-1}\right] \tag{44}$$

$$=\left[\varphi^2\left(2^{-4K_{it}^y}\sigma_y^2+\left(1-2^{-2K_{it}^y}\right)^2\nu_y^2\right)+\left(2^{-4K_{it}^a}\sigma_{ai}^2+\left(1-2^{-2K_{it}^a}\right)^2\nu_{ai}^2\right)\right] \tag{45}$$

$$= \left[\varphi^2 \left(2^{-4K_{it}^y}\sigma_y^2 + \frac{\left(1-2^{-2K_{it}^y}\right)^2}{2^{2K_{it}^y}-1}\sigma_y^2 \right) + \left(2^{-4K_{it}^a}\sigma_{ai}^2 + \frac{\left(1-2^{-2K_{it}^a}\right)^2}{2^{2K_{it}^a}-1}\sigma_{ai}^2 \right) \right] \tag{46}$$

$$= \left[\varphi^2 \left(\frac{1-2^{-2K_{it}^y}}{2^{2K_{it}^y}-1} \right) \sigma_y^2 + \left(\frac{1-2^{-2K_{it}^a}}{2^{2K_{it}^a}-1} \right) \sigma_a^2 \right] \tag{47}$$

$$= \varphi^2 2^{-2K_{it}^y}\sigma_y^2 + 2^{-2K_{it}^a}\sigma_a^2 \tag{48}$$

$$= \varphi^2 2^{-2K_{it}^y}\sigma_b^2 + 2^{-2K_{it}^a}\sigma_a^2 \tag{49}$$

The objective function becomes

$$\min_{K_{it}^y} \varphi^2 2^{-2K_{it}^y}\sigma_b^2 + 2^{-2(K-K_{it}^y)}\sigma_a^2 \tag{50}$$

first-order conditions:

$$\varphi^2(-2)\ln(2)2^{-2K_{it}^{y*}}\sigma_b^2 + 2\ln(2)2^{-2(K-K_{it}^{y*})}\sigma_a^2 = 0 \tag{51}$$

$$\varphi^2 2^{-2K_{it}^{y*}}\sigma_b^2 = 2^{-2(K-K_{it}^{y*})}\sigma_a^2 \tag{52}$$

taking $\log_{(2)}$ of everything:

$$-2K_{it}^{y*} + \log_2(\varphi^2\sigma_b^2) = -2K_{it} + 2K_{it}^{y*} + \log_2 \sigma_a^2 \tag{53}$$

$$K_{it}^{y*} = \frac{1}{4}\log_2\left(\varphi^2\frac{\sigma_b^2}{\sigma_a^2}\right) + \frac{1}{2}K \tag{54}$$

$$K_{it}^{y*} = \frac{1}{2}\log_2\left(\varphi\frac{\sigma_b}{\sigma_a}\right) + \frac{1}{2}K \tag{55}$$

B.2 Comparative Statics: Disagreement

Using the perceived volatility of the tracking noises and optimal attention allocation

$$\nu_y^2 = \frac{1}{2^{2K_{it}^y}-1}\sigma_y^2, \quad K_{it}^{y*} = \frac{1}{4}\log_2\left(\varphi^2\frac{\sigma_b^2}{\sigma_a^2}\right) + \frac{1}{2}K$$

Differentiating it with respect to σ_b^2:

$$\frac{d\nu_y^2}{d\sigma_b^2} = \frac{1}{2^{2K_{it}^y}-1} + \sigma_b^2\frac{d}{dK_{it}^y}\left(\frac{1}{2^{2K_{it}^y}-1}\right)\frac{dK_{it}^y}{d\sigma_b^2}$$

where

$$\frac{d}{dK_{it}^{y}}\left(\frac{1}{2^{2K_{it}^{y}}-1}\right)=\frac{(-2)\ln(2)2^{2K_{it}^{y}}}{(2^{2K_{it}^{y}}-1)^2}$$

$$\frac{d}{d\sigma_b^2}\left(\frac{1}{4}\log_2\left(\varphi^2\frac{\sigma_b^2}{\sigma_a^2}\right)+\frac{1}{2}K\right)=\frac{1}{4}\frac{1}{\sigma_b^2\ln(2)}$$

therefore,

$$\begin{aligned}\frac{d\nu_y^2}{d\sigma_b^2}&=\frac{1}{2^{2K_{it}^{y}}-1}+\sigma_b^2\frac{(-2)\ln(2)2^{2K_{it}^{y}}}{(2^{2K_{it}^{y}}-1)^2}\frac{1}{4}\frac{1}{\sigma_b^2\ln(2)}\\&=\frac{1}{2^{2K_{it}^{y}}-1}-\frac{1}{2}\frac{2^{2K_{it}^{y}}}{(2^{2K_{it}^{y}}-1)^2}\\&=\frac{2(2^{2K_{it}^{y}}-1)-2^{2K_{it}^{y}}}{2(2^{2K_{it}^{y}}-1)^2}\\&=\frac{-2+2^{2K_{it}^{y}}}{2(2^{2K_{it}^{y}}-1)^2}\gtrless 0\end{aligned}$$

Differentiating it with respect to K and σ_a^2 results in:

$$\begin{aligned}\frac{d\nu_y^2}{dK}&=\frac{d\nu_y^2}{dK_{it}^{y}}\frac{dK_{it}^{y}}{dK}\\&=\sigma_b^2(-1)\frac{d2^{2K_{it}^{y}}}{dK_{it}^{y}}\left(\frac{1}{2^{2K_{it}^{y}}-1}\right)^2\frac{dK_{it}^{y}}{dK}\\&=\sigma_b^2(-1)2\ln(2)2^{2K_{it}^{y}}\left(\frac{1}{2^{2K_{it}^{y}}-1}\right)^2\frac{dK_{it}^{y}}{dK}\\&=-\sigma_b^2 2\ln(2)2^{2K_{it}^{y}}\left(\frac{1}{2^{2K_{it}^{y}}-1}\right)^2\frac{1}{2}\\&=-\sigma_b^2\ln(2)2^{2K_{it}^{y}}\left(\frac{1}{2^{2K_{it}^{y}}-1}\right)^2<0\end{aligned}$$

$$\begin{aligned}\frac{d\nu_y^2}{d\sigma_a^2}&=\frac{d\nu_y^2}{dK_{it}^{y}}\frac{dK_{it}^{y}}{d\sigma_a^2}\\&=\sigma_b^2(-1)2\ln(2)2^{2K_{it}^{y}}\left(\frac{1}{2^{2K_{it}^{y}}-1}\right)^2\frac{dK_{it}^{y}}{d\sigma_a^2}\\&=\sigma_b^2(-1)2\ln(2)2^{2K_{it}^{y}}\left(\frac{1}{2^{2K_{it}^{y}}-1}\right)^2\left(-\frac{1}{4\ln(2)\sigma_a^2}\right)\end{aligned}$$

$$= \frac{1}{2}\frac{\sigma_b^2}{\sigma_a^2} 2^{2K_{it}^y} \left(\frac{1}{2^{2K_{it}^y} - 1} \right)^2 > 0$$

B.3 Comparative Statics: Price Setting

To show how do prices respond to monetary shocks under different conditions, we combine the pricing rule p_{it} and signal structure s_{it}^y:

$$p_{it} = \varphi \left(1 - 2^{-2K_{it}^y}\right) s_{it}^y - \left(1 - 2^{-2K_{it}^a}\right) s_{it}^a \tag{56}$$

where

$$s_{it}^y = y_t + it^y = b_t - cr_t + it^y$$

$$\frac{ds_{it}^y}{dr_t} = -c \tag{57}$$

$$\frac{dp_{it}}{dr_t} = \frac{dp_{it}}{ds_{it}^y}\frac{ds_{it}^y}{dr_t} = \varphi \left(1 - 2^{-2K_{it}^y}\right)(-c) < 0 \tag{58}$$

Which means that as r $\uparrow$, $p_{it} \downarrow$

Then, we can replace $2^{-2K_{it}^y}$ using

$$K_{it}^{y*} = \frac{1}{2}\log_2\left(\varphi \frac{\sigma_b}{\sigma_a}\right) + \frac{1}{2}K$$

such that

$$2^{-2K_{it}^y} = \frac{\sigma_a}{\sigma_b \varphi} 2^{-K}$$

Rewriting it we arrive at:

$$\frac{dp_{it}}{dr_t} = \left(1 - 2^{-2K_{it}^y}\right)(-c) = -\varphi c \left(1 - \frac{\sigma_a}{\sigma_b\varphi}2^{-K} < 0\right)$$

Taking the second-order comparative statics with respect to K_{it}, σ_a and σ_b:

$$\frac{d}{dK_{it}}\left(\frac{dp_{it}}{dr_t}\right) = -\ln(2)\frac{\sigma_a}{\sigma_b\varphi}2^{-K}\varphi c < 0 \tag{59}$$

$$\frac{d}{d\sigma_a}\left(\frac{dp_{it}}{dr_t}\right) = \frac{1}{\sigma_b\varphi}2^{-K}\varphi c > 0 \tag{60}$$

$$\frac{d}{d\sigma_b}\left(\frac{dp_{it}}{dr_t}\right) = \frac{\sigma_a}{\varphi}(-1)\frac{1}{\sigma_b^2}2^{-K}\varphi c < 0 \tag{61}$$