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Across the borders, above the bounds: a non-linear framework for international yield curves

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Abstract

This paper presents a non-linear framework to evaluate spillovers across domestic and international yield curves when policy rates are constrained by the zero lower bound. Based on the sample of US and UK data, we estimate a joint shadow rate model of international yield curves, accounting for the zero lower bound, no-arbitrage conditions within and between government bond markets, and the global nature of some of the bond risk factors. Results indicate that the post-2009 US monetary policy transmission mechanism and its spillover effects on the UK yield curve are non-linear and asymmetric.

Key words: Joint term structure models, local projections, monetary policy, non-linear responses, shadow rate term structure models, yield curve, zero lower bound.

JEL classification: E43, E47, E52, G15.

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1 Introduction

This paper presents a framework for evaluating monetary policy spillovers across domestic and international yield curves when policy rates are constrained by the zero lower bound (ZLB).

Monetary policy transmission through interest rates and its spillover effects is an old and well-studied question. From the vast and prominent literature, we know that monetary policy has an influence on entire yield curves (see e.g., among many, Rigobon and Sack 2004, Kuttner 2001, Gürkaynak, Sack and Swanson 2005), that it affects yield curves via interest rate expectations and also via term premia (Hanson and Stein 2015, Gertler and Karadi 2015, Kaminska, Mumtaz and Šustek 2021), and, moreover, that these impacts are not restricted to domestic yield curves, especially in the case of US monetary policy surprises (see Craine and Martin 2008, Hausman and Wongswan 2011). More recently, when policy rates reached the ZLB and the largest central banks switched from the conventional policy of setting short interest rates to unconventional monetary policies, the question has seen renewed interest and new stylised facts were established. Most studies find significant unconventional monetary policy effects on the yields of long-term government bonds,[1] while the US policy is also found to be effective in reducing international bond yields (e.g. Bauer and Neely 2014), with substantially increased spillovers post-2009 (e.g. Albagli, Ceballos, Claro and Romero 2019). However, the question of how the monetary policy shock transmission is affected by the presence of the interest rate ZLB is still unaddressed.

This paper contributes to the debate by presenting a non-linear framework to evaluate monetary policy spillovers when interest rates are constrained by the ZLB. Importantly, the framework allows us to derive non-linear yield curve responses to monetary policy shocks.

[1]See Dell'Ariccia, Rabanal and Sandri (2018) and Busetto, Chavaz, Froemel, Joyce, Kaminska and Worlidge (2022) for the overview

As a result, we are able to confirm the large international role of US monetary policy, and, what is new, we also show that the effects of domestic and international monetary policy transmission are not symmetric and depend significantly on the closeness to the ZLB.

Specifically, we propose to analyse the monetary policy transmission through the lenses of a joint no-arbitrage term structure model of foreign and domestic interest rates. The model assumes that bond markets are integrated (as in Greenwood, Hanson, Stein and Sunderam 2019, Gourinchas, Ray and Vayanos 2022) and explicitly imposes the ZLB on interest rates. This unified approach to jointly exploit information in international bond markets has several advantages. First, the joint model provides a consistent approach across interest rates, and it helps us to account for the global nature of bond risk factors,[2] and to analyse jointly movements in domestic and foreign policy rate expectations and term premia. Second, it allows us to explicitly account for the ZLB, which is crucial for estimating plausible (i.e. non-negative) expectations for policy rates over the sample. Third, it allows us to take into account the non-linearities that emerge at the ZLB.

The empirical relevance of the ZLB constraint and the importance of accounting for it when carrying out inference about interest rates and monetary policy near the ZLB have been well recognised in the term structure literature (see, for instance, Christensen and Rudebusch 2015, Bauer and Rudebusch 2016). This implies that the standard Gaussian Affine Term Structure Models (GATSM), which are usually employed to analyse the term premia and expected rate components of long term rates, are not fit to study the ZLB period, as they do not impose the non-negativity restrictions. Therefore, to better understand the monetary policy spillovers and their channels post 2009, we extend the so-called "shadow rate term structure model" (SRTSM) framework,[3] which explicitly imposes ZLB restrictions

[2]See, for example, Diebold, Li and Yue (2008) and Coroneo, Garrett and Sanhueza (2018) on global yield factors being important drivers of country bond yields

[3]For instance, Krippner (2015), Wu and Xia (2016), Bauer and Rudebusch (2016).

on the individual term structure models, and apply it to modelling jointly international yield curves.

We estimate the joint SRTSM on the interest rates derived from UK and US government bonds from October 1992 to December 2019. We then use our joint shadow rate model to assess the US monetary policy transmission mechanism and its spillover effects on the UK yield curve. In particular, we use the local projection approach introduced by Jordà (2005) to estimate the responses of the latent state variables, and then feed these responses into the joint shadow rate model to obtain the (non-linear) responses of forward rates.

Our main findings are as follows. First, we find that US bond factors operate as global factors and account for a significant proportion of the variation of UK bond yields. We also find evidence that, in order to explain UK-specific yield curve movements, a local factor is required. Second, we demonstrate that explicitly imposing ZLB restrictions on interest rates is key for the model performance and tractability: the restricted joint SRTSM model outperforms its unrestricted Gaussian counterpart (GATSM) in delivering improved yield curve fit. Importantly, the SRTSM specification turns out to be crucial for explaining the increased correlation between US and UK longer maturity rates post 2009 (see e.g. Roberts-Sklar 2015).

Finally, we show that the post-2009 US monetary policy transmission mechanism and its spillover effects on the UK yield curve are non-linear and asymmetric. When close to the ZLB, a target US monetary policy shock is transmitted to short rates quicker in case of tightening than easing. Moreover, while increasing the shock magnitude in case of tightening delivers a proportionally increased response in interest rates, the transmission impact is far from proportional in the case of easing, with rates undershooting at shorter horizons.

Taken together, these results indicate that, while the US monetary policy and other global factors are key in shaping the dynamics of US and UK yield curves, to analyse these dynamics and to evaluate the transmission and spillover of shocks post the global financial crisis of 2007-2008, it is important to account for the ZLB on interest rates and the non-linearities caused by it.

The remainder of the paper is organized as follows. Section 2 reviews the recent literature analysing domestic and international bond yield curves during the low policy rate environment. The term structure model is presented in Section 3, while the econometric methodology and the data are described in Sections 4 and 5 correspondingly. We subsequently present the main findings in Section 6. Finally, Section 7 concludes our analysis.

2 Literature Review

A monetary policy tightening in the US or an intensification of geopolitical tensions can have a significant impact on interest rates in a small open economy. This sensitivity of the yield curves to the global factors limits the influence of domestic policies on financial conditions and hence presents a challenge to the monetary policies across advanced and emerging economies.

Given the significance of the issue, it is not surprising that a large body of empirical literature has been dedicated to studying the effects of global factors, and especially of US monetary policy, on domestic and international yield curves. Thus, before presenting our framework, we briefly overview the available evidence and discuss our contribution to the literature.

The joint framework for international yield curves and the importance of global factors has been mainly analysed by the empirical macroeconomics literature (e.g. Diebold et al. 2008, Kumar and Okimoto 2011, Del Negro, Giannone, Giannoni and Tambalotti 2019, Coroneo et al. 2018). This literature finds that global and US factors tend to explain a large share of the variance of domestic yields. It also shows that the response of yields to shocks to global factors is substantial and long-lasting.

Papers that estimate global effects on international bond prices while imposing the discipline of no-arbitrage (like Sarno, Schneider and Wagner 2012, Kaminska, Meldrum and Smith 2013, Jotikasthira, Le and Lundblad 2015, Gourinchas et al. 2022) deliver similar key messages on the importance of global and US factors. However, the no-arbitrage framework presents the benefit of decomposition of yields into expectations and premia and hence allows for a more structural analysis of the global and local factor impacts on domestic and international yield curves. Our paper belongs most closely to this strand of the literature.

What distinguishes us from the no-arbitrage literature studying the international yield curves jointly is the ZLB and the implied non-linearity of yields in bond factors. To impose the ZLB in a joint framework we extend the so-called "shadow rate term structure model" framework (see e.g. Krippner 2015, Wu and Xia 2016, Bauer and Rudebusch 2016), which explicitly imposes ZLB restrictions on the individual term structure models, and apply it to a joint model of US and UK yield curves. In particular, we build on and extend the Wu and Xia (2016) approach.

The implied non-linear joint yield curve framework allows us to analyse the US monetary policy shocks transmission. In studying this question, we are close to Craine and Martin (2008), Albagli et al. (2019), Miranda-Agrippino and Rey (2020), Degasperi, Hong and Ricco (2020), Kearns, Schrimpf and Xia (2018) and Gourinchas et al. (2022), among many others. However, the available empirical studies are mostly reduced-form and analyse the shock transmission via linear methods. Instead, we believe to be the first to study this question in the no-arbitrage framework with the explicit ZLB constrain imposed. This approach delivers non-linear functions for shock responses, which turns out to be key for understanding the nature of monetary policy shock transmission post-2009.

3 Term structure model

We jointly model the term structure of interest rates on US and UK government bonds. We take the point of view of a US-based investor and impose no-arbitrage restrictions across bonds denominated in US dollars and UK sterling. The rest of this section provides a brief overview of the term structure model.

3.1 Setup

We assume that interest rates depend on n_0 common factors $\boldsymbol{x}_t^0$, related to global interest rate trends, and of n_i country-specific factors $\boldsymbol{x}_t^i$, driven by local trends. Let $n = n_0 + n_{US} + n_{UK}$, and denote the full $n \times 1$ vector of state variables as

$$\boldsymbol{x}_t = \left(\boldsymbol{x}_t^{0\prime}, \boldsymbol{x}_t^{US\prime}, \boldsymbol{x}_t^{UK\prime}\right)'.$$

We assume that the $(n \times 1)$ vector of state variables follows a first order Gaussian vector autoregressive process under the physical measure $\mathbb{P}$:

$$\boldsymbol{x}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}\boldsymbol{x}_t + \boldsymbol{\Gamma}\boldsymbol{\varepsilon}_{t+1}, \tag{1}$$

where $\boldsymbol{\varepsilon}_{t+1} \overset{\mathbb{P}}{\sim} \mathcal{N}_{IID}(\mathbf{0}, \mathbf{I}_n)$ and $\boldsymbol{\Gamma}$ is a $(n \times n)$ lower triangular matrix. We also assume that $\boldsymbol{\Phi}$ is lower triangular, and that $\boldsymbol{x}_t^{US}$ is independent from $\boldsymbol{x}_t^{UK}$ conditionally on $\boldsymbol{x}_t^0$; in other words, the $(n_{US} + n_{UK}) \times (n_{US} + n_{UK})$ lower right block of $\boldsymbol{\Phi}$ is lower triangular and block diagonal. This assumption makes the joint term structure model *decomposable* (see Egorov et al., 2011, pp. 60-61 for a discussion) under the historical measure $\mathbb{P}$, i.e., it can be decomposed into two single country term structure models.

To define the dynamics of the state variables under the risk neutral measures, it is convenient to define:

$$\boldsymbol{z}_t^{US} = \left(\boldsymbol{x}_t^{0\prime}, \boldsymbol{x}_t^{US\prime}\right)^\prime \quad \text{and} \quad \boldsymbol{z}_t^{UK} = \left(\boldsymbol{x}_t^{0\prime}, \boldsymbol{x}_t^{UK\prime}\right)^\prime,$$

two vectors with $m_{US} = n_0 + n_{US}$ and $m_{UK} = n_0 + n_{UK}$ elements, respectively. We also define $\boldsymbol{\varepsilon}_t^{US}$ and $\boldsymbol{\varepsilon}_t^{UK}$ as two vectors obtained by partitioning $\boldsymbol{\varepsilon}_t$ conformably with $\boldsymbol{z}_t^{US}$ and $\boldsymbol{z}_t^{UK}$, and similarly for $\boldsymbol{\mu}^{US}$ and $\boldsymbol{\mu}^{UK}$, $\boldsymbol{\Phi}^{US}$ and $\boldsymbol{\Phi}^{UK}$, $\boldsymbol{\Gamma}^{US}$ and $\boldsymbol{\Gamma}^{UK}$. Notice that, because of decomposability assumption, $\boldsymbol{z}_t^{US}$ and $\boldsymbol{z}_t^{UK}$ also follow under $\mathbb{P}$ a Gaussian first order autoregressive process.

To impose decomposability under the risk-neutral measures, we assume for each country an essentially affine stochastic discount factor as in Duffee (2002):

$$M_{t+1}^i = \exp\left(-r_t^i - \frac{1}{2}\boldsymbol{\lambda}_t^{i\prime}\boldsymbol{\lambda}_t^i - \boldsymbol{\lambda}_t^{i\prime}\boldsymbol{\varepsilon}_{t+1}^i\right), \quad i = US, UK,$$

where $\boldsymbol{\lambda}_t^i$ is the price of risk in country i, affine in the factors that affect the term structure in that country:

$$\boldsymbol{\lambda}_t^i = \boldsymbol{\lambda}^i + \boldsymbol{\Lambda}^i \boldsymbol{z}_t^i, \quad i = US, UK.$$

Notice that the price of risk in country i is a vector of m_i elements, $i = US, UK$. The assumptions above imply that the dynamics of the factors under the risk neutral measures $\mathbb{Q}^i$, $i = US, UK$, are also VAR(1):

$$\boldsymbol{z}_{t+1}^i = \boldsymbol{\mu}^{\mathbb{Q}^i} + \boldsymbol{\Phi}^{\mathbb{Q}^i}\boldsymbol{z}_t^i + \boldsymbol{\Gamma}^i\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}^i}, \quad i = US, UK, \tag{2}$$

where $\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}^i} \overset{\mathbb{Q}^i}{\sim} \mathcal{N}_{IID}(\mathbf{0}, \mathbf{I}_{m_i})$ and the parameters under $\mathbb{P}$ and $\mathbb{Q}^i$ are related by:

$$\boldsymbol{\mu}^{\mathbb{Q}^i} = \boldsymbol{\mu}^i - \boldsymbol{\Gamma}^i \boldsymbol{\lambda}^i \quad \text{and} \quad \boldsymbol{\Phi}^{\mathbb{Q}^i} = \boldsymbol{\Phi}^i - \boldsymbol{\Gamma}^i \boldsymbol{\Lambda}^i, \quad \text{for } i = UK, US$$

Notice that whereas $\boldsymbol{\mu}$ is a n-vector, $\boldsymbol{\mu}^{\mathbb{Q}^i}$ and $\boldsymbol{\lambda}^i$ are two m_i-vectors; similarly, while $\boldsymbol{\Phi}$ and $\boldsymbol{\Gamma}$ are two $(n \times n)$-matrices, $\boldsymbol{\Phi}^{\mathbb{Q}^i}$, $\boldsymbol{\Gamma}^i$ and $\boldsymbol{\Lambda}^i$ are $(m_i \times m_i)$ matrices.

We also assume that the country i's short term interest rate, r_t^i, does not depend on the local factors of country j, $\boldsymbol{x}_t^j$, and vice versa; in turn, this implies that the shape of the term structure of interest rates in country i at date t only depends on $\boldsymbol{z}_t^i$, and not on $\boldsymbol{x}_t^j$. This makes our joint term structure model decomposable also under the risk-neutral measure for each country, and it allows us to estimate the term structure model in each country without using information from the other country.

3.2 Interest rates

Shadow short rates can be expressed as

$$s_t^i = \delta_0^i + \boldsymbol{\delta}_1^{i\prime} \boldsymbol{z}_t^i, \quad i = US, UK. \tag{3}$$

The lower bound on observed short-term interest rates can be enforced by allowing each country's observed rate to be equal to the corresponding shadow short rate only when the latter is above the lower bound, and otherwise equal to the lower bound:

$$r_t^i = \max(s_t^i, \underline{r}^i), \quad i = US, UK. \tag{4}$$

Under absence of arbitrage, the price $p_{t,\tau}^i$ of a zero-coupon bond with τ months to maturity in country i can be expressed as

$$p^i_{t,\tau} = E^{\mathbb{Q}^i}_t \left[\exp\left(-\sum_{j=0}^{\tau-1} r^i_{t+j} \right) \right], \quad i = US, UK.$$

The assumption in (4) implies that yields are nonlinear in state variables and do not have an analytical expression. Denote by $f^i_{t,\tau}$ the time t one period forward rate in country i for a loan starting at $t+\tau$. Wu and Xia (2016) show that, under (2), (3) and (4), the forward rate $f^i_{t,\tau}$ is approximately equal to:

$$f^i_{t,\tau} \approx \underline{r}^i + \sigma^i_\tau g\left(\frac{a^i_\tau + \boldsymbol{b}^{i\prime}_\tau \boldsymbol{z}^i_t - \underline{r}^i}{\sigma^i_\tau} \right) \equiv h^i_\tau(\boldsymbol{x}_t), \quad i = US, UK, \tag{5}$$

where:

$$\begin{aligned}
g(w) &= wN(w) + n(w), \\
\boldsymbol{b}^i_\tau &= \left[\left(\boldsymbol{\Phi}^{\mathbb{Q}^i} \right)^\tau \right]' \boldsymbol{\delta}^i_1, \\
a^i_\tau &= \delta^i_0 + \left(\sum_{j=0}^{\tau-1} \boldsymbol{b}^i_j \right)' \boldsymbol{\mu}^{\mathbb{Q}^i} - \frac{1}{2} \left(\sum_{j=0}^{\tau-1} \boldsymbol{b}^i_j \right)' \boldsymbol{\Gamma}\boldsymbol{\Gamma}' \left(\sum_{j=0}^{\tau-1} \boldsymbol{b}^i_j \right), \\
\left(\sigma^i_\tau\right)^2 &= \sum_{j=0}^{\tau-1} \boldsymbol{b}^{i\prime}_j \boldsymbol{\Gamma}\boldsymbol{\Gamma}' \boldsymbol{b}^i_j,
\end{aligned} \tag{6}$$

with $N(\cdot)$ and $n(\cdot)$ denoting the cdf and the pdf of the standard normal distribution, respectively.

3.3 Exchange rates

Under the assumption of no arbitrage across bonds denominated in different currencies, our model also provides the exchange rate implications.

Let e_t be the log spot sterling-dollar exchange rate, i.e. the spot price of sterling in units of dollars. Assuming that both markets are frictionless for foreign and domestic investors, the one-period return on domestic and foreign bonds must be equal. Under complete markets and absence of arbitrage, Backus, Foresi and Telmer (2001) have shown that

$$\begin{aligned} e_t - e_{t-1} &= \log M_t^{UK} - \log M_t^{US} \\ &= \left(r_{t-1}^{US} - r_{t-1}^{UK}\right) + \frac{1}{2}\left(\boldsymbol{\lambda}_{t-1}^{US\prime}\boldsymbol{\lambda}_{t-1}^{US} - \boldsymbol{\lambda}_{t-1}^{UK\prime}\boldsymbol{\lambda}_{t-1}^{UK}\right) + \boldsymbol{\lambda}_{t-1}^{US\prime}\boldsymbol{\varepsilon}_t^{US} - \boldsymbol{\lambda}_{t-1}^{UK\prime}\boldsymbol{\varepsilon}_t^{UK}, \end{aligned} \tag{7}$$

where, from equation (1):

$$\begin{aligned} \boldsymbol{\varepsilon}_t^{US} &= \left(\boldsymbol{\Gamma}^{US}\right)^{-1}(\boldsymbol{z}_t^{US} - \boldsymbol{\mu}^{US} - \boldsymbol{\Phi}^{US}\boldsymbol{z}_{t-1}^{US}) \\ \boldsymbol{\varepsilon}_t^{UK} &= \left(\boldsymbol{\Gamma}^{UK}\right)^{-1}(\boldsymbol{z}_t^{UK} - \boldsymbol{\mu}^{UK} - \boldsymbol{\Phi}^{UK}\boldsymbol{z}_{t-1}^{UK}). \end{aligned}$$

This allows us to state the measurement equation of the depreciation rate (7) as a function of $\boldsymbol{x}_t$ and $\boldsymbol{x}_{t-1}$:

$$e_t - e_{t-1} = k(\boldsymbol{x}_t, \boldsymbol{x}_{t-1})$$

where $k(\cdot,\cdot)$ is defined by (7).

4 Inference

4.1 State-space representation

The joint shadow rate term structure model introduced in the section above can be presented in a state-space form. The input data sample consists of panels of forward rates for the two countries, $f_{t,\tau}^{US}$ and $f_{t,\tau}^{UK}$, $t = 1, \ldots, T$ and $\tau = \tau_1, \ldots, \tau_K$, and of the time series of the end-of-month sterling-dollar exchange rate e_t, $t = 1, \ldots, T$. In the joint term structure model, the space equation is a system of observation equations for forward rates and a sterling-dollar depreciation exchange rate.

Let us collect the observed forward rates in:

$$\boldsymbol{f}_t = (\boldsymbol{f}_t^{US\prime}, \boldsymbol{f}_t^{UK\prime})',$$

with $\boldsymbol{f}_t^{US} = (f_{t,\tau_1}^{US}, \ldots, f_{t,\tau_K}^{US})'$ and $\boldsymbol{f}_t^{UK} = (f_{t,\tau_1}^{UK}, \ldots, f_{t,\tau_K}^{UK})'$, and the model implied forward rates in:

$$\boldsymbol{h}(\boldsymbol{x}_t) = [\boldsymbol{h}^{US}(\boldsymbol{x}_t)', \boldsymbol{h}^{UK}(\boldsymbol{x}_t)']',$$

with $\boldsymbol{h}^{US}(\boldsymbol{x}_t) = [h_{\tau_1}^{US}(\boldsymbol{x}_t), \ldots, h_{\tau_K}^{US}(\boldsymbol{x}_t]'$ and $\boldsymbol{h}^{UK}(\boldsymbol{x}_t) = [h_{\tau_1}^{UK}(\boldsymbol{x}_t), \ldots, h_{\tau_K}^{UK}(\boldsymbol{x}_t]'$. The elements of $\boldsymbol{h}(\boldsymbol{x}_t)$ were defined in (5).

We collect $(2K + 1)$ measurement equations in:

$$\boldsymbol{y}_t = \boldsymbol{m}(\boldsymbol{x}_t, \boldsymbol{x}_{t-1}) + \boldsymbol{u}_t,$$

where

$$\boldsymbol{y}_t = \begin{pmatrix} \boldsymbol{f}_t \\ \Delta e_t \end{pmatrix}, \quad \boldsymbol{m}(\boldsymbol{x}_t, \boldsymbol{x}_{t-1}) = \begin{pmatrix} \boldsymbol{h}(\boldsymbol{x}_t) \\ k(\boldsymbol{x}_t, \boldsymbol{x}_{t-1}) \end{pmatrix}, \tag{8}$$

and where the measurement error $\boldsymbol{u}_t$ is IID Normally distributed with mean zero and variance $\boldsymbol{\Omega}$. We assume that measurement errors are uncorrelated (across rates, countries and maturities) and with variance that depends on the country but not on the forward rate maturity. The measurement error standard deviations ω^{US}, ω^{UK} and ω^e must be estimated with the other parameters.

Since the depreciation rate is function of both $\boldsymbol{x}_t$ and $\boldsymbol{x}_{t-1}$, we need to define the state vector as

$$\boldsymbol{\xi}_t = (\boldsymbol{x}'_t, \boldsymbol{x}'_{t-1})',$$

which allows us to rewrite the measurement equation as

$$\boldsymbol{y}_t = \boldsymbol{m}(\boldsymbol{\xi}_t) + \boldsymbol{u}_t. \tag{9}$$

The state, or transition, equation is then given by

$$\boldsymbol{\xi}_t = \boldsymbol{\mu}_\xi + \boldsymbol{\Phi}_\xi \boldsymbol{\xi}_{t-1} + \boldsymbol{v}_t \tag{10}$$

where $\boldsymbol{v}_t \sim \mathcal{N}_{IID}(\mathbf{0}, \boldsymbol{\Sigma}_\xi)$, and

$$\boldsymbol{\mu}_\xi = \begin{pmatrix} \boldsymbol{\mu} \\ \mathbf{0}_{n\times 1} \end{pmatrix}, \quad \boldsymbol{\Phi}_\xi = \begin{pmatrix} \boldsymbol{\Phi} & \mathbf{0}_{n\times n} \\ \mathbf{I}_n & \mathbf{0}_{n\times n} \end{pmatrix}, \quad \boldsymbol{\Sigma}_\xi = \begin{pmatrix} \boldsymbol{\Sigma} & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \end{pmatrix}, \tag{11}$$

and $\boldsymbol{\Sigma} = \boldsymbol{\Gamma}\boldsymbol{\Gamma}'$.

Although the state equation is linear in the state variables, the space, or measurement, equation in (9) is nonlinear. Therefore, we base quasi maximum likelihood inference on the Extended Kalman filter (EKF).

4.2 Identification

To uniquely identify the latent states, we use an identification scheme similar to Dai and Singleton (2000) and Egorov, Li and Ng (2011), and impose $\boldsymbol{\Gamma} = \mathbf{I}_n$, $\boldsymbol{\delta}_1^{US} \geq 0$, $\boldsymbol{\delta}_1^{UK} \geq 0$ and $\boldsymbol{\mu} = \mathbf{0}_{n\times 1}$. Also, as mentioned above, $\boldsymbol{\Phi}$ is lower diagonal with a lower right block structure, whereas $\boldsymbol{\Phi}^{\mathbb{Q}^{US}}$ and $\boldsymbol{\Phi}^{\mathbb{Q}^{UK}}$ are full. This implies that the full parameter vector $\boldsymbol{\theta}$ is given by:

- δ_0^{US}, δ_0^{UK} (2 parameters),
- $\boldsymbol{\delta}_1^{US}$, $\boldsymbol{\delta}_1^{UK}$ ($m_{US} + m_{UK}$ parameters),
- $\boldsymbol{\Phi}^{\mathbb{Q}^i}$, $i = US, UK$ ($m_{US}^2 + m_{UK}^2$ parameters),
- $\boldsymbol{\mu}^{\mathbb{Q}^i}$, $i = US, UK$ ($m_{US} + m_{UK}$ parameters),
- $\boldsymbol{\Phi}$ ($n(n+1)/2 - n_{US}n_{UK}$ parameters),
- ω^{US}, ω^{UK}, ω^{e} (3 parameters)

assuming that $\underline{r}^{US} = \underline{r}^{UK} = 0$. Recall that $m_{US} = n_0 + n_{US}$ and $m_{UK} = n_0 + n_{UK}$. In total, there are $(10 + 5n_0^2 + 5n_{UK} + 3n_{UK}^2 + 5n_{US} + 3n_{US}^2 + n_0(9 + 6n_{UK} + 6n_{US}))/2$ parameters.

We impose stationarity under $\mathbb{P}$ by constraining the diagonal elements of $\boldsymbol{\Phi}$ between 0 and 1; we also impose stationarity under $\mathbb{Q}^i$ by imposing that all the eigenvalues of each $\boldsymbol{\Phi}^{\mathbb{Q}}i$ are strictly smaller than 1.

4.3 Shock responses at the zero lower bound

We now use the joint shadow rate model to assess the impact of an exogenous shock. We use the local projection approach introduced by Jordà (2005) to estimate the responses of the latent factors to exogenous shocks, and then feed these responses into the joint shadow rate model to obtain the responses of forward rates and forward premia.

We start by projecting each factor h steps ahead (for $h = 0, \ldots, H$) on a lag of all the factors and on an exogenous shock g_t

$$x_{j,t+h} = \alpha_{j,h} + \boldsymbol{\gamma}_{j,h}\boldsymbol{x}_{t-1} + \beta_{j,h} g_t + \nu_{j,t+h}, \quad j = 1, \ldots, n. \tag{12}$$

We then compute the conditional expectations of the factors as

$$\begin{aligned}
\boldsymbol{x}_{t+h|g_t=1} &= E(\boldsymbol{x}_{t+h}|g_t=1) = \boldsymbol{\alpha}_h + \boldsymbol{\gamma}_h \boldsymbol{x}_{t-1} + \boldsymbol{\beta}_h && (13)\\
\boldsymbol{x}_{t+h|g_t=0} &= E(\boldsymbol{x}_{t+h}|g_t=0) = \boldsymbol{\alpha}_h + \boldsymbol{\gamma}_h \boldsymbol{x}_{t-1} && (14)
\end{aligned}$$

and the factor response at horizon h as

$$\Delta \boldsymbol{x}_{t+h|g_t} = \boldsymbol{x}_{t+h|g_t=1} - \boldsymbol{x}_{t+h|g_t=0} = \boldsymbol{\beta}_h \tag{15}$$

This is the standard impulse response used in the literature, however in our case we are not interested in the responses of the state variables but in the ones of the observables. In order to do so, we first compute the response of the shadow short rate

$$\Delta s^i_{t+h|g_t} = s^i_{t+h|g_t=1} - s^i_{t+h|g_t=0} = \boldsymbol{\delta}_1^{i'} \Delta \boldsymbol{x}_{t+h|g_t} = \boldsymbol{\delta}_1^{i'} \boldsymbol{\beta}_h, \quad i = US, UK. \tag{16}$$

and then feed these responses into the short-rate equation

$$\begin{aligned}
\Delta r^i_{t+h|g_t} &= r^i_{t+h|g_t=1} - r^i_{t+h|g_t=0} \\
&= \max\{s^i_{t+h|g_t=1}, 0\} - \max\{s^i_{t+h|g_t=0}, 0\} \\
&= \max\{\boldsymbol{\delta}_0^{i'} + \boldsymbol{\delta}_1^{i'} \boldsymbol{x}_{t+h|g_t=1}, 0\} - \max\{\boldsymbol{\delta}_0^{i'} + \boldsymbol{\delta}_1^{i'} \boldsymbol{x}_{t+h|g_t=0}, 0\}, \quad i = US, UK. && (17)
\end{aligned}$$

Here we see the effect of the ZLB. If the short rate is at the ZLB, its response to a monetary policy easing is constrained. Also, when the shock pushes the short rate to the ZLB, its response becomes muted. On the other hand, when the shock lifts off the shadow short rate from the ZLB, the short rate becomes responsive.

In the same way, we can compute the responses of the forward rates to the shock

$$\Delta \boldsymbol{f}^i_{t+h,\tau|g_t} = \boldsymbol{f}^i_{t+h,\tau|g_t=1} - \boldsymbol{f}^i_{t+h,\tau|g_t=0} \approx \boldsymbol{h}^i_\tau(\boldsymbol{x}_{t+h|g_t=1}) - \boldsymbol{h}^i_\tau(\boldsymbol{x}_{t+h|g_t=0}), \quad i = US, UK. \tag{18}$$

Notice that these responses are time-varying, as they depend on the distance of the shadow short rate from the ZLB at the time of the shock, and therefore they depend also on the size of the shock, as a larger monetary policy easing shock can bring the shadow short rate above the ZLB.

We estimate the local projection equation (12) via LS and we compute confidence intervals by block-bootstrapping the residuals in (12) with the overlapping stationary circular block-bootstrap of Politis and Romano (1994) with maximum block size equal to the minimum between $h+1$ and $2\lfloor T^{1/4} \rfloor$, and 500 replications.

Given that the responses of observed rates in equations (17)-(18) are conditional on $\boldsymbol{x}_{t+h|g_t=0}$, which represents the trajectory of the state variables from the time of the shock to h periods later in the absence of a shock, we adopt a "scenario" approach in which we condition on the realised state variables. Therefore the estimated responses represent the additional effect on observed rates due to the considered shock.

5 Data and preliminary evidence

We use 1-month forward rates on US and UK government bonds for maturities of 3 and 6 months, 1, 3, 5 and 10 years. The sample consists of end-of-month observations from October 1992 to December 2019.[4] US rates are constructed as in Wu and Xia (2016) using

[4] To ensure the internal consistency of the model, we assume a constant lower bound. Therefore, we end the sample in 2019, i.e. before the Bank of England Monetary Policy Committee started considering the possibility of negative policy rates, see Monetary Policy Committee (2020).

daily yields available on the Federal Reserve Board web page. UK rates are also constructed as in Wu and Xia (2016) from monthly yields available on the Bank of England website (3 and 6 month maturity) and from Bank of England calculations based on Bloomberg data (for the other maturities). We also use end-of-month data on the dollar-sterling exchange rate from the FRED data set.

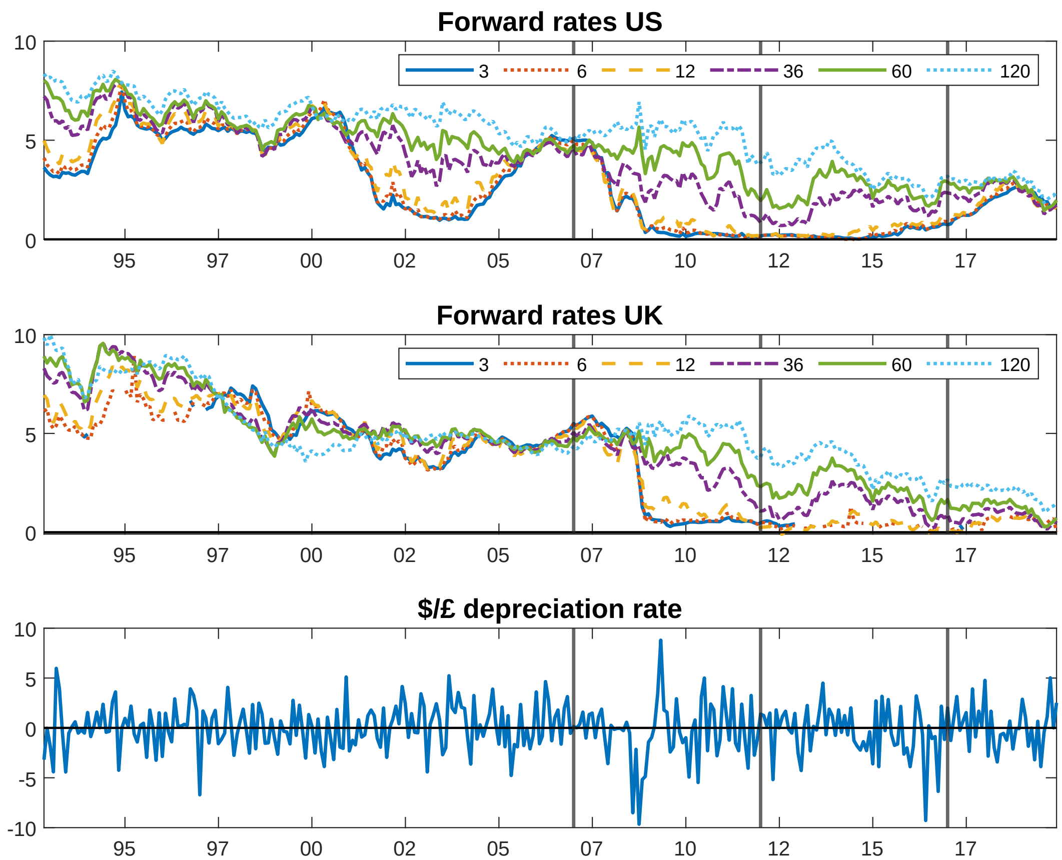
Figure 1 reports the data used, and shows how the level of short-term interest rates declined towards the ZLB at the end of 2008 and has been close to this bound after this date, especially for the UK. The figure also reveals that for the UK there are missing values for the 3 and 6 month rates at the beginning and at the end of the sample.

The three vertical lines in the figure indicate three specific dates in our sample, representing different states of monetary policy: the 1st refers to January 2007, when US and UK interest rates were well above the ZLB; the 2nd refers to January 2012, when the Federal Funds rate was at the ZLB with little space for further decreases, while the UK short rate was still above the ZLB; the 3rd refers to January 2017, when the Federal Funds rate was above the ZLB, while the UK short rate was at the ZLB.

The joint shadow rate term structure model in Section 3 allows the US and the UK yield curves to be driven by common and country-specific factors. To underline the importance of the common factors, Figure 2 shows the striking co-movement between UK and US forward rates at long maturities. Focussing separately on before and after 2009, it shows that the comovement between longer maturity forward rates increased after 2009 compared to the pre-ZLB period.

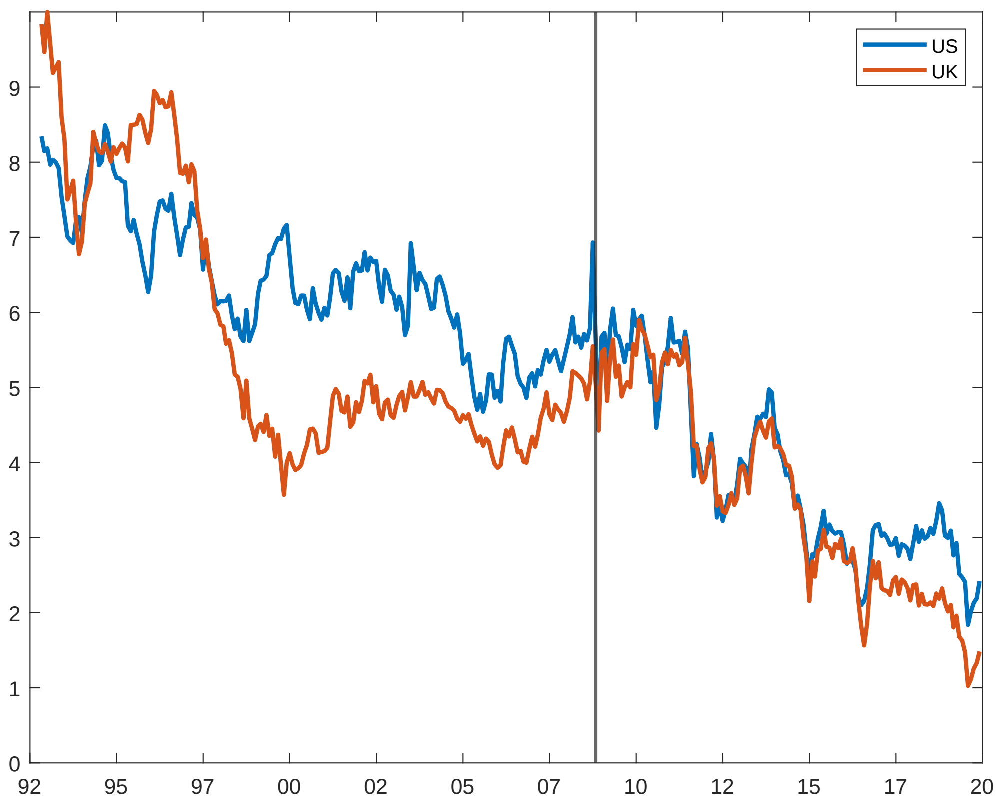
To determine the number of each type of factors, we perform principal component analysis, and we start by analyzing each forward curve separately. The first two columns of Table 1 report the cumulative variance of US and UK forward rates explained by the corresponding first five principal components (PCs) extracted separately for each country. The

Figure 1: Data



Note: This figure presents the forward rates and exchange rates used in the estimation. The sample period is from October 1992 to December 2019; data are at monthly frequency. The top two figures show the observed term structure of instantaneous forward rates for the 3-, 6-, 12-, 36-, 60- and 120-month maturities; the bottom figure shows the monthly depreciation rate of the US dollar – UK pound sterling exchange rate. The three vertical lines indicate three specific dates in our sample, representing different states of monetary policy: the 1st refers to January 2007, when US and UK interest rates were far from the lower bound; the 2nd refers to January 2012, when the Federal Funds rate was almost at the ELB with little space for further decreases, while the UK short rate was not constrained by the lower bound; the 3rd refers to January 2017, when the Federal Funds rate was above the ELB, while the UK short rate was at the lower bound. Sources: Bank of England, Bloomberg Finance L.P., Federal Reserve Board, FRED data set.

Figure 2: 10-year forward rates in US and UK



Note: The figure shows the instantaneous US and UK forward rates for the 120-month maturities at monthly frequency. The sample period ranges from October 1992 to December 2019. The vertical line denotes December 2008. Sources: Bank of England, Bloomberg Finance L.P., Federal Reserve Board.

table indicates that the first three country PCs explain most of each country forward curve variance (99.8% for both the US and the UK).

We then pool the two forward curves and extract PCs jointly. The cumulative joint variance of US and UK forward rates explained by these joint PCs is reported in the last column of Table 1. The table shows that four joint PCs are required to explain at least 99.1% of the joint variation in US and UK interest rates. This indicates that while each country forward curve is well described by 3 factors, in accordance with much of the term

Table 1: Cumulative proportion of variance explained by Principal Components

	US rates	UK rates	All rates
PC1	0.871	0.893	0.857
PC2	0.989	0.990	0.960
PC3	0.998	0.998	0.979
PC4	1.000	0.999	0.993
PC5	1.000	1.000	0.997

Note: Average cumulative proportion of variance of US rates (first column), UK rates (second column), and all rates (third column) explained by the first five PCs extracted from US rates (first column), UK rates (second column), and jointly from US and UK rates (third column).

structure literature starting from Litterman and Scheinkman (1991), once we pool the two forward curves together, we need to allow for an additional factor.

Given the size and the importance of the US Treasury market, we use the US factors to proxy for the common factors and we analyze how much of the variation in UK rates and the first three UK PCs is explained by the US PCs. In Table 2, we report the cumulative proportion of the variance of UK rates and UK PCs explained by US PCs. The table indicates that three US PCs explain a large proportion of the variance of UK rates (89.1%). This because the first UK PC is almost perfectly explained by the first US PC and the second UK PC is partially explained by the second US PC. However, the third UK PC seems to be less related to the third US PC.

Overall, Tables 1-2 suggest that four factors are needed to explain the two forward curves; three of these factors are common US factors, and one is specific to the UK. Accordingly, in specifying our model we choose $n = 4$, , $n_0 = 3$, $n_{US} = 0$ and $n_{UK} = 1$.[5]

[5] In Appendix B, we show that results from our specification are robust to an alternative specification with two global factors, one US-specific factor and one UK-specific factor, i.e. $n = 4$, $n_0 = 2$, $n_{US} = 1$ and $n_{UK} = 1$.

Table 2: Cumulative proportion of variance of UK rates explained by US PCs

# US PCs	UK rates	UK PC1	UK PC2	UK PC3
1	0.795	0.886	0.030	0.013
First 2	0.883	0.939	0.456	0.031
First 3	0.891	0.946	0.456	0.258
First 4	0.892	0.946	0.456	0.248
First 5	0.903	0.956	0.475	0.320

Note: cumulative proportion of variance of UK rates and UK PCs explained by US PCs.

6 Results

6.1 Model fit

We estimate the joint shadow rate term structure model described in Section 3 by quasi maximum likelihood (QML) as described in Section 4. Estimated parameters reported in Table 3 indicate that all the factors are heavily persistent under both the physical and the risk-neutral measures. The lagged first factor has a significant effect on the second factor in both the physical and the two risk-neutral measures. In addition, under the UK risk-neutral measure we have that also the lagged UK factor has a significant effect on the second common factor, indicating feedback from the UK forward curve to the US one. The UK factor is significantly affected by all the common US factors, under both the UK risk-neutral and the physical measures. Finally, the estimated variance of the measurement errors is the same for both the US and the UK forward curves, indicating that the joint shadow rate model equally fits the two forward curves ($\omega^{UK}=\omega^{US}$), while instead the estimated variance of the measurement error for the depreciation rate is larger ($\omega^{e}=0.0217 > \omega^{UK}=0.0019$).

Root mean squared errors (RMSEs) reported in Table 4 confirm that the average full sample fit of the two yield curves is comparable. However, looking at individual maturities

Table 3: Parameter estimates

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
δ_0^{US}	0.0740**	$\Phi_{11}^{Q^{UK}}$	0.9742**	$\mu_3^{Q^{US}}$	−0.2711
δ_0^{UK}	0.0849**	$\Phi_{12}^{Q^{UK}}$	−0.0010	$\mu_1^{Q^{UK}}$	0.0428
$\delta_{1,1}^{US}$	0.0090**	$\Phi_{13}^{Q^{UK}}$	0.0076	$\mu_2^{Q^{UK}}$	0.4656**
$\delta_{1,2}^{US}$	0.0017	$\Phi_{14}^{Q^{UK}}$	−0.0037**	$\mu_3^{Q^{UK}}$	−0.2625
$\delta_{1,3}^{US}$	0.0034	$\Phi_{21}^{Q^{UK}}$	0.0083**	$\mu_4^{Q^{UK}}$	0.0011
$\delta_{1,1}^{UK}$	0.0085**	$\Phi_{22}^{Q^{UK}}$	0.9950**	Φ_{11}	0.9999**
$\delta_{1,2}^{UK}$	0.0039**	$\Phi_{23}^{Q^{UK}}$	−0.0020	Φ_{21}	0.0326*
$\delta_{1,3}^{UK}$	0.0061**	$\Phi_{24}^{Q^{UK}}$	−0.0018**	Φ_{22}	0.9965**
$\delta_{1,4}^{UK}$	0.0007**	$\Phi_{31}^{Q^{UK}}$	0.0032	Φ_{31}	0.0043
$\Phi_{11}^{Q^{US}}$	0.9751**	$\Phi_{32}^{Q^{UK}}$	0.0005	Φ_{32}	0.0012
$\Phi_{12}^{Q^{US}}$	0.0027	$\Phi_{33}^{Q^{UK}}$	0.9312**	Φ_{33}	0.9425**
$\Phi_{13}^{Q^{US}}$	−0.0007	$\Phi_{34}^{Q^{UK}}$	−0.0013	Φ_{41}	0.0102**
$\Phi_{21}^{Q^{US}}$	0.0109**	$\Phi_{41}^{Q^{UK}}$	0.0091**	Φ_{42}	0.0037**
$\Phi_{22}^{Q^{US}}$	0.9981**	$\Phi_{42}^{Q^{UK}}$	0.0033**	Φ_{43}	0.0065*
$\Phi_{23}^{Q^{US}}$	−0.0022	$\Phi_{43}^{Q^{UK}}$	0.0052*	Φ_{44}	0.9996**
$\Phi_{31}^{Q^{US}}$	0.0041	$\Phi_{44}^{Q^{UK}}$	0.9993**	ω^{US}	0.0019**
$\Phi_{32}^{Q^{US}}$	0.0016	$\mu_1^{Q^{US}}$	0.0376	ω^{UK}	0.0019**
$\Phi_{33}^{Q^{US}}$	0.9257**	$\mu_2^{Q^{US}}$	0.4742**	ω^e	0.0217**

Note: this table reports the estimated parameters of the joint shadow rate term structure model for US and UK forward rates, and the depreciation rate. * and ** denote significance at the 5% and 1% level using using the QML sandwich formula to compute asymptotic standard errors.

we can see that the 10-year US forward is the worst fit maturity for the US yield curve, and the 3-year UK forward is the worst fit maturity for the UK yield curve. The table also reports the fit in two sub-samples: the first sub-sample from October 1992 to November 2008 (when interest rates were far from the ZLB) and the second sub-sample from December 2008 to December 2019 (when interest rates were close or at the ZLB). For the US, the fit across the two sub-samples is broadly comparable, while for the UK the model fits better in the ZLB sub-sample.

Table 4: Joint shadow rate model fit

	US			UK		
	Full	Pre-LB	LB	Full	Pre-LB	LB
3	0.164	0.168	0.157	0.167	0.179	0.132
6	0.110	0.083	0.141	0.146	0.156	0.125
12	0.138	0.149	0.121	0.161	0.185	0.116
36	0.176	0.185	0.161	0.213	0.219	0.205
60	0.164	0.170	0.155	0.144	0.154	0.127
120	0.251	0.220	0.291	0.165	0.177	0.146
av	0.167	0.163	0.171	0.166	0.179	0.142

Note: this table reports RMSEs for the joint shadow rate model for US and UK rates, left and right panel respectively. Results are reported for the full sample (Oct 1992 - Dec 2019), the pre-lower bound sample (Oct 1992 - Nov 2008) and the lower bound sample (Dec 2008 - Dec 2019).

To understand the role of the ZLB and of the depreciation rate in our joint shadow rate model for US and UK forward rates and the depreciation rate (SRM with FX), we consider three additional models:

- A joint shadow rate model for only US and UK forward rates (SRM w/o FX). This can be implemented in our framework by imposing that the vector of observables $\boldsymbol{y}_t$ in (8) has the last element (corresponding to the depreciation rate) missing.

- A joint Gaussian term structure model for US and UK forward rates and the depreciation rate (GM with FX). This model does not explicitly impose a ZLB on forward rates, and can be seen as a particular case of the joint shadow rate model in which $\underline{r}^i = -\infty$ in (4), implying that $r^i_t = s^i_t$.

- A joint Gaussian model for only US and UK forward rates (GM w/o FX) that has both the last element of $\boldsymbol{y}_t$ in (8) missing and $\underline{r}^i = -\infty$ in (4).

Table 5 reports RMSEs of the four models for the full sample and the ZLB sub-sample.

Table 5: Interest rates and depreciation rate fit

	Interest rates		Depreciation rate	
	Full	LB	Full	LB
SRM with FX	0.167	0.156	1.462	1.169
SRM w/o FX	0.170	0.157	-	-
GM with FX	0.180	0.170	1.654	1.361
GM w/o FX	0.178	0.162	-	-

Note: this table reports RMSEs for interest rates (first two columns) and the depreciation rate (last two columns) from four model specifications: 1) the joint shadow rate model with the depreciation rate (SRM with FX), 2) the joint shadow rate model without the depreciation rate (SRM w/o FX), 3) the joint Gaussian model with the depreciation rate (GM with FX), and 4) the joint Gaussian model without the depreciation rate (GM w/o FX). Interest rates RMSEs refer to averages across maturities. Results are reported for the full sample (Oct 1992 - Dec 2019) and the lower bound sample (Dec 2008 - Dec 2019).

Results indicate that our baseline joint shadow rate model with FX achieves the best fit for interest rates, followed by the joint shadow rate model with only forward rates. The two Gaussian models have larger RMSEs for interest rates, and in the lower bound period the Gaussian model with the depreciation rate has larger RMSEs than the model for forward rates only. The fit for the depreciation rate is overall less accurate, but also in this case the baseline joint shadow rate model delivers the best fit. This indicates that the superior fit for interest rates in the joint shadow rate model with the depreciation rate is not obtained by sacrificing the fit of the depreciation rate, as the joint shadow rate model with FX better fits the joint behaviour of the depreciation rate and forward rates. Thus, accounting for the interest rate ZLB matters also for the fit of the FX depreciation rates.

Finally, Table 6 shows that "SRM with FX" specification performs best in capturing the correlation between US and UK long-term rates: the model implied correlations are closest to those observed in the data consistently across sub-samples, while Gaussian models

Table 6: Correlation 10y rates

	Full	Pre-LB	LB
Data	0.873	0.792	0.966
SRM with FX	0.887	0.809	0.963
SRM w/o FX	0.885	0.811	0.973
GM with FX	0.888	0.840	0.933
GM w/o FX	0.899	0.874	0.936

Note: this table reports the correlation between the 10 year US rate and the 10 year UK rate in the data (first row) and the model implied ones. Results are reported for the full sample (Oct 1992 - Dec 2019), the pre-lower bound sample (Oct 1992 - Nov 2008) and the lower bound sample (Dec 2008 - Dec 2019).

overestimate the correlation prior the ELB, and underestimates it afterwards. In sum, only the model with the shadow rates can explain the stylised fact of significantly increased correlation between US and UK longer maturity rates post 2009. This further justifies the model choice.

Overall, the results presented in this subsection indicate that, due to the non-linearity introduced by the ZLB, the shadow rate model presents a more accurate description of US and UK interest rates than the Gaussian model, both before and during the lower bound period. On the other hand, the contribution of the depreciation rate seems marginal.[6]

6.2 Responses to US monetary policy shocks

This section analyses the transmission of US monetary policy surprises. For this exercise, we use conventional monetary policy shocks of Kaminska et al. (2021), constructed using a high-frequency yield curve decomposition around FOMC announcements.

We start with the action, or so-called "target" shock, associated with a change in the

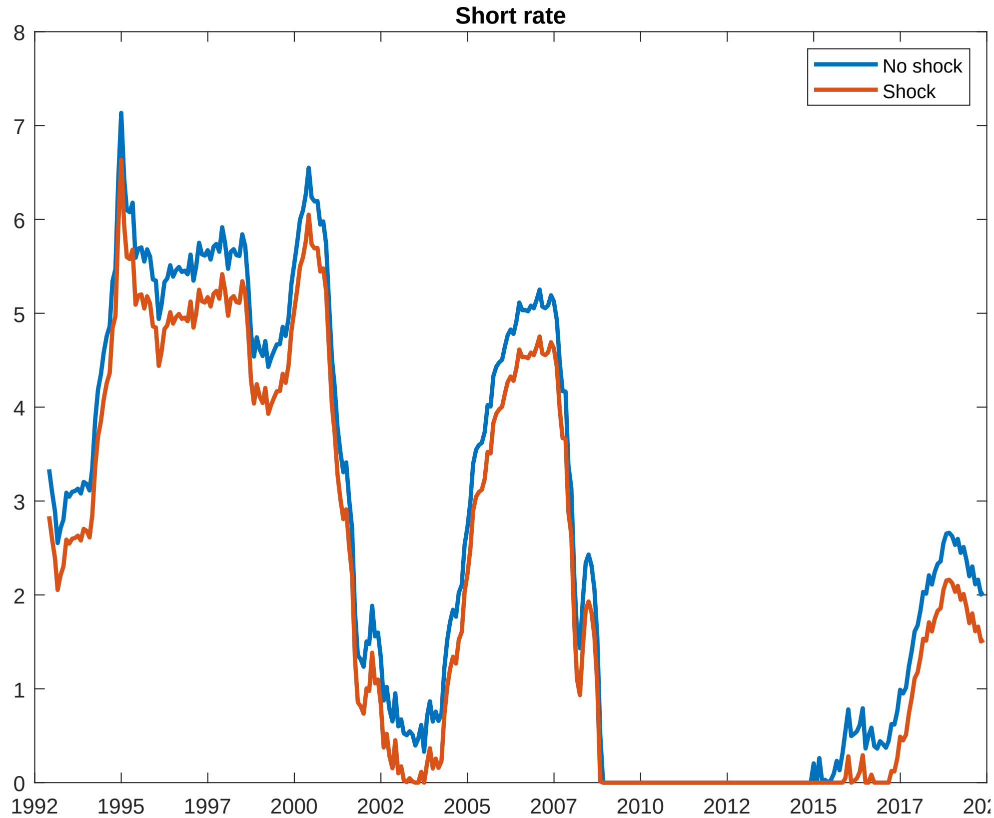
[6] In Appendix C we show that the results presented in the paper are robust to excluding the depreciation rate from the model.

current policy rate. Figure 3 reports the effect of a -50bp US target shock on the US short rate on impact.[7] The blue line is the conditional expectation of the short rate without shock ($\widehat{r}_{t|g_t=0}$) and the red line is the conditional expectation of the short rate with the shock ($\widehat{r}_{t|g_t=1}$). The distance between the two lines is the effect of the shock on the short rate on impact ($h = 0$). As it can be seen from the figure, this is equal to -50bp most of the times (due to our normalization), but when the short rate reaches the ZLB, the impact response is muted.

The first panel of Figure 4 reports the responses of the US and UK shadow short rates to a -50bp shock to the target of US rates at different horizons. The plot indicates that the shock has indeed a long-lasting impact on the US shadow short rate. The figure also indicates a spillover effect of the US monetary policy shock to the UK shadow rate, the effect is smaller in size but significant in the first 12 months. However, shadow short rates are observable only when interest rates are away from the ZLB, short rates instead are always observable. For this reason, in the other three plots in Figure 4, we report the impulse responses of US and UK short rates on the three dates indicated in Figure 1 with vertical lines. The top right panel refers to January 2007, when interest rates in both countries were well above the ZLB. In this case, the responses of the short rates are equal to the response of the shadow short rates for both countries. However, this is not the case for the two bottom plots. In January 2012, the Federal Funds rate was at only 8bp with little space for further decreases, therefore the shock effect on the US short rate is muted in the first seven months after the shock. The response becomes significantly negative nine months after the shock, but eventually the response to the shock brings back the short rate to the ZLB after a bit more than two years. On the other hand, the responses of the UK short rate are not constrained by the lower bound and thus equal to the ones of the shadow short rate. The bottom right

[7] In practice, the shock has been normalised to have -50bp effect on the short rate in normal times.

Figure 3: Reponse of US short rate on impact

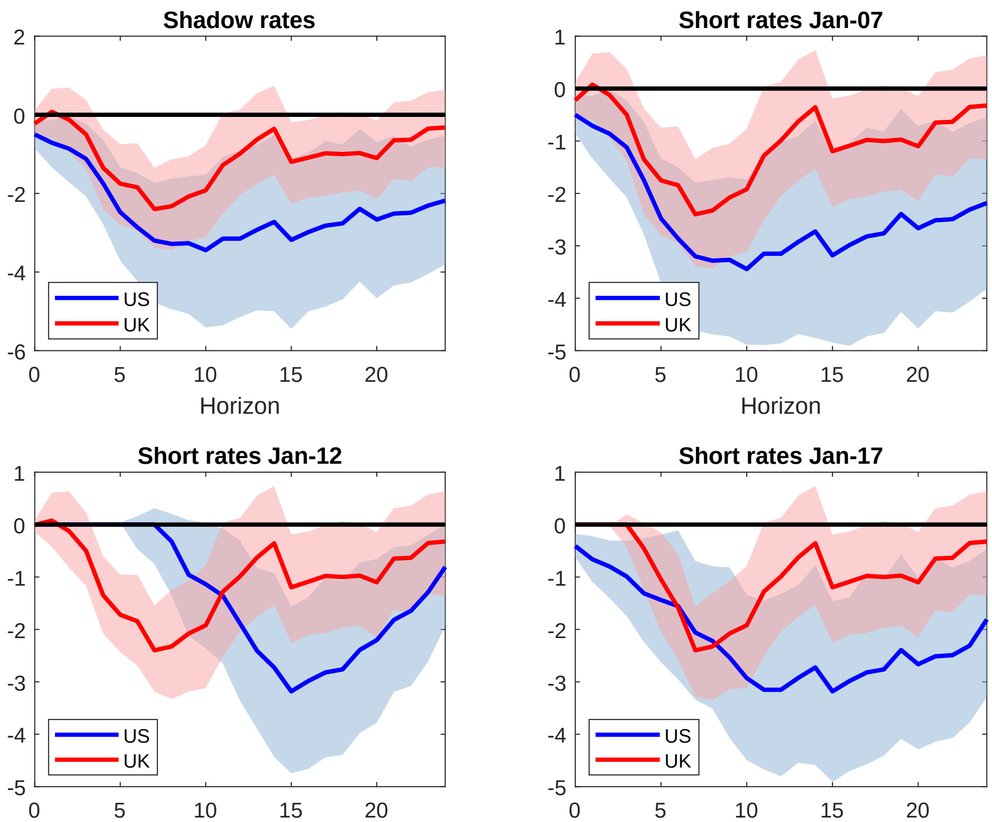


Impact responses of the US short rate to a -50bp path shock of Kaminska et al. (2021). The blue line is the estimated $\widehat{r}_{t|g_t=0}$, i.e. the conditional expectation of the short rate without shock, and the red line is $\widehat{r}_{t|g_t=1}$, i.e. the conditional expectation of the short rate with the shock. The distance between the two lines is the effect of the shock on the short rate on impact ($h = 0$). The shock has been normalised to have -50bp impact effect on the shadow rate, i.e. on the short rate in normal times.

plot refers to January 2017, when the Federal Funds rate was at 65bp, so it could fully accommodate a target monetary policy easing with a -50bp impact. Indeed the response of the US short rate is almost unconstrained. The UK short rate is at the ZLB at this date, so the response to the shock is muted for the first months, but then it becomes unconstrained again and significantly negative up to the first eleven months after the shock.

Of course, the interest rate lower bound affects shorter maturities more than long ones.

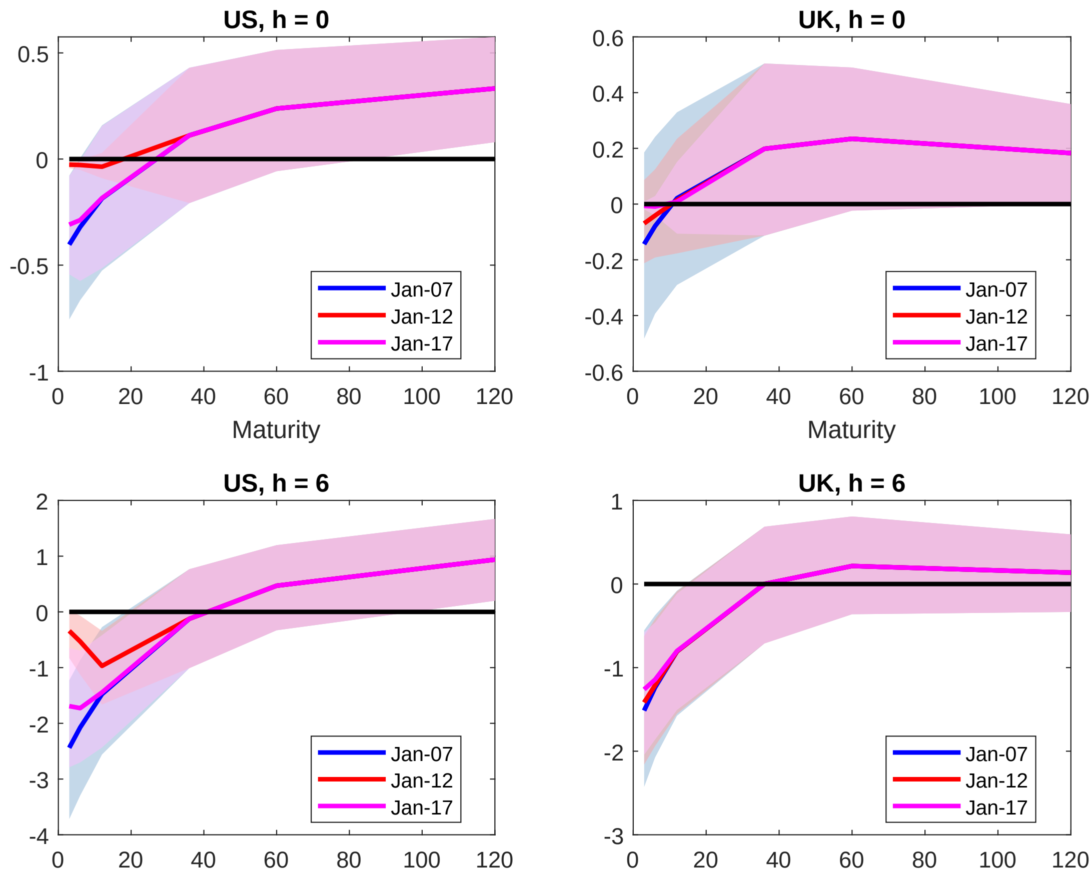
Figure 4: Shadow short rate and short rate responses to -50bp US target shock



Impulse responses of the shadow short rate (top left plot) and the short rates on three dates (Jan 2007, Jan 2012 and Jan 2017) to a -50bp target shock of Kaminska et al. (2021). The shock has been normalised to have -50bp impact effect on the short rate in normal times. The shaded areas indicate the 80% confidence intervals.

However, as shown in Coroneo and Pastorello (2020), the effect of the non-linearity can be empirically relevant also for long-term interest rates. To investigate this possibility, in Figure 5 we report the responses of all the US and UK forward rates in our sample on impact ($h = 0$) and six months after the shock ($h = 6$). The results suggest that while the non-linearity of responses to the policy shock depends on the proximity to the ZLB and is important for short rates, it is less of an issue for maturities longer than 3 years. In fact, the forward rates responses for maturities 36 months and longer do not seem to be affected by the ZLB.

Figure 5: Forward curve response to -50bp US target shock



Impulse responses of US (left plots) and UK (right plots) forward curves on three dates (Jan 2007, Jan 2012 and Jan 2017) to a -50bp target shock of Kaminska et al. (2021). The top plots report the impact responses ($h = 0$), and the bottom plots the responses after 6 months ($h = 6$). The shock has been normalised to have -50bp impact effect on the short rate in normal times. The shaded areas indicate the 80% confidence intervals.

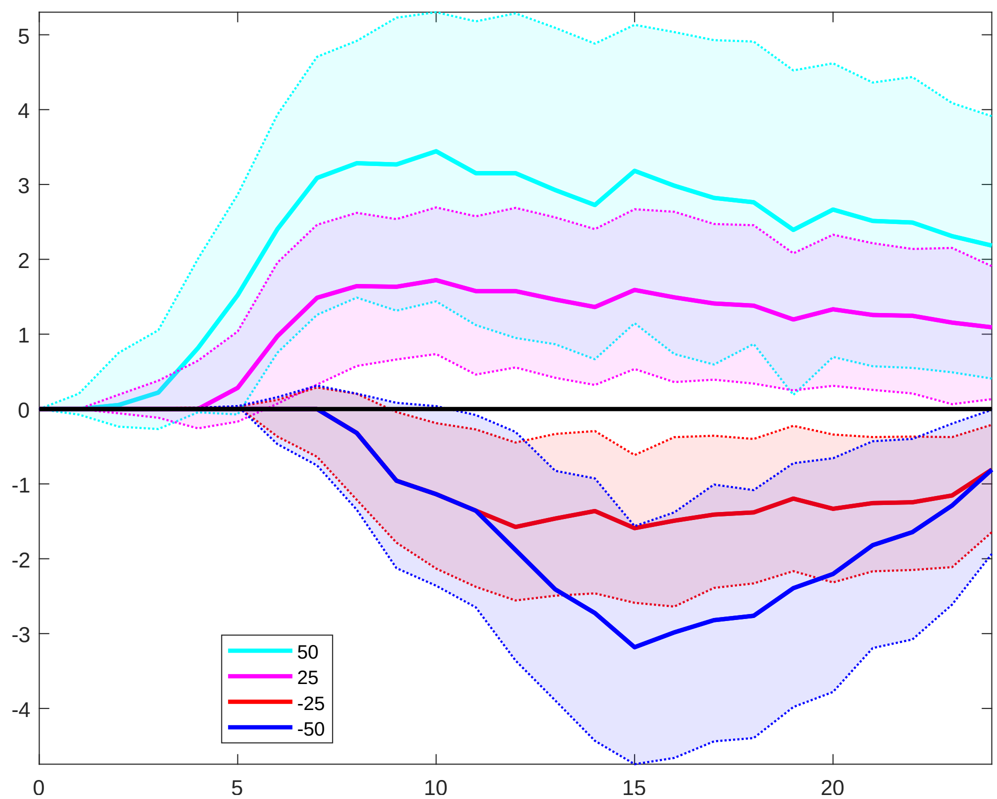
All results so far assume a negative shock that has an impact effect on the US short shadow rate of -50bp, but, due to the non-linearities of the model, the shape of the responses depends on the sign and size of the shock. When rates are at the ZLB responses to negative shocks can be muted or constrained, but less so to positive shocks. Also, a large negative shock is more likely to bring rates close to the ZLB than a small negative shock. For this reason, in Figure 6, we report the US short rate responses on Jan 2012 to a target shocks with impact effect on the shadow short rate of a 50pb, 25bp, -25bp and a -50bp. On this date, the US short rate was at the ZLB and whatever the sign and magnitude of the shock, the impact effect on the short rate is muted, but at longer horizons we see that a large positive shock lifts-off the short rate before a smaller one. Also, negative shocks take more time to impact the short rate, and the shape of the responses depends on the size of the shock, especially for negative ones.

Finally, Figure 7, contains responses of the US and UK forward curves to a US path shock.[8] This shock is associated with a response of interest rate expectations to FOMC announcements and is consistent with both the Fed information effect (Nakamura and Steinsson 2018) and the Fed response to news channel (Bauer and Swanson 2020). The figure indicates that the response of both the US and the UK forward curve is markedly different from the responses to the target shock. In particular, the path shock significantly shifts US forward rates with maturities larger than a year by the same amount. For UK forward rates instead the effect is significant only for mid-maturities. In any case, we do not find any evidence of important non-linearities for the path shock, as it mainly affects mid-long rates.

In conclusion, our analysis highlights two important results. First, the implied responses from the shadow rate model of mid-short rates to a US target shock are highly non-linear at the ZLB, as their shape depends on both the sign and the size of the shock. Second, we find significant spillovers of US monetary policy shocks to UK rates. In particular, a target

[8]In this application, we normalised the path shock to have an impact response on the US 36-month forward rate of -25bp in normal times

Figure 6: US short rate responses to a US target shock on Jan 2012



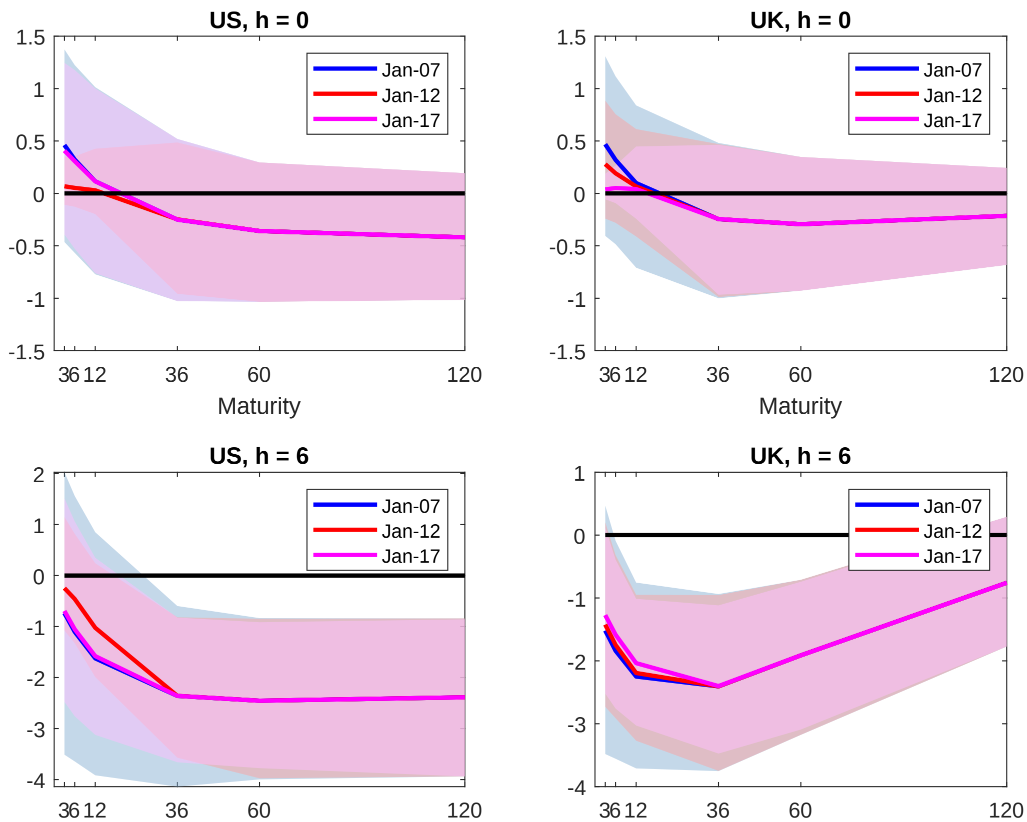
Impulse responses of the US short rate on Jan 2012 to a target shock of Kaminska et al. (2021) with impact effect on the shadow short rate of a 50pb, 25bp, -25bp and a -50bp. The shaded areas indicate the 80% confidence intervals.

policy easing significantly decreases UK short rates in the first 12 months, and a path shock significantly affects UK mid rates at a six-month horizon.

7 Conclusions

This paper advances our understanding of domestic and international transmission of monetary policy shocks. To study the shock spillovers, we adopt a framework that incorporates

Figure 7: Forward curve response to -25bp US path shock



Impulse responses of forward rates on three dates (Jan 2007, Jan 2012 and Jan 2017) to a -25bp path shock of Kaminska et al. (2021). The shock is normalised to have an impact response of the US 36-month forward rate of -25bp in normal times. The shaded areas indicate the 80% confidence intervals.

three important elements. First, we explicitly account for the ZLB on policy rates, and hence depart from the standard linear frameworks, which have been usually employed by the existing works studying the policy transmission.

Second, we introduce a joint term structure model of the domestic and international forwad curves, which provides us with a useful and rigorous tool to study the shock transmission via forward curves.

Third, we use a novel approach that combines linear local projection and the joint shadow rate term structure model to analyse responses of interest rates to US monetary policy shocks and their spillover effects on UK yields. The nonlinearity implied by the ZLB turns out to be key for policy transmission at home and abroad. In fact, we show that the US monetary policy transmission mechanism and its spillover effects on the UK forward curve are highly non-linear and asymmetric. When close to the ZLB, a target monetary policy shock is transmitted quicker in case of tightening than easing. Notably, the importance of non-linearities introduced by the ZLB is only evident for shorter maturity rates, while forward rates with maturities three years or longer are largely unaffected by the distance to the ZLB.

The focus of our analysis is the transmission of conventional monetary policy. However, the question of asymmetries and non-linearities becomes even more important when analysing the transmission of unconventional monetary policies, such as Quantitative Easing (QE) and Quantitative Tightening (QT), the effects of which are less well established than that of policy rate. While QT remains relatively new, several advanced economies have now begun the process of unwinding their balance sheets. The empirical impact of QT has been especially hard to measure so far, given the very recent nature of QE unwind, but it is widely believed that the impact is likely to be asymmetric. Once the sample capturing the QT period is sufficiently large, our approach should provide a useful tool to revisit the QT impact and assess the importance of the ZLB for the presumed asymmetric nature of the unconventional monetary policies.

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A Implementing the Extended Kalman Filter

- We use a diffuse initialization of the Extended Kalman filter as follows:

$$\widehat{\boldsymbol{\xi}}_{1|0} = \mathrm{E}(\boldsymbol{\xi}_1) = (\mathbf{I}_{2n} - \boldsymbol{\Phi}_{\xi})^{-1}\boldsymbol{\mu}_{\xi}$$

  and

$$\mathbf{P}_{1|0} = \mathrm{E}[(\boldsymbol{\xi}_1 - \widehat{\boldsymbol{\xi}}_1)(\boldsymbol{\xi}_1 - \widehat{\boldsymbol{\xi}}_1)'] = 100\,\mathbf{I}_{2n}.$$

  Notice that, using (11), the initial state value can be rewritten as:

$$\widehat{\boldsymbol{\xi}}_{1|0} = \begin{bmatrix} (\mathbf{I}_n - \boldsymbol{\Phi})^{-1}\boldsymbol{\mu} \\ (\mathbf{I}_n - \boldsymbol{\Phi})^{-1}\boldsymbol{\mu} \end{bmatrix}.$$

- The forecasts of the observables' values and their approximate MSEs are given by:

$$\widehat{\boldsymbol{y}}_{t|t-1} = \boldsymbol{m}(\widehat{\boldsymbol{\xi}}_{t|t-1}), \quad t = 1, \dots, T$$

  and

$$\mathrm{E}[(\boldsymbol{y}_t - \widehat{\boldsymbol{y}}_{t|t-1})(\boldsymbol{y}_t - \widehat{\boldsymbol{y}}_{t|t-1})'] \approx \widehat{\mathbf{M}}'_{t|t-1}\mathbf{P}_{t|t-1}\widehat{\mathbf{M}}_{t|t-1} + \boldsymbol{\Omega}, \quad t = 1, \dots, T,$$

  where $\widehat{\mathbf{M}}_{t|t-1} = \mathbf{M}(\widehat{\boldsymbol{\xi}}_{t|t-1}) = \left.\dfrac{\partial \boldsymbol{m}(\boldsymbol{\xi}_t)'}{\partial \boldsymbol{\xi}_t}\right|_{\boldsymbol{\xi}_t = \widehat{\boldsymbol{\xi}}_{t|t-1}}$ is the Jacobian matrix of the model implied forward rates and depreciation rates. Its expression is as follows: given (8), we can partition $\mathbf{M}(\boldsymbol{\xi}_t)$ as:

$$\mathbf{M}(\boldsymbol{\xi}_t) = \begin{bmatrix} \mathbf{H}^{US}(\boldsymbol{x}_t) & \mathbf{H}^{UK}(\boldsymbol{x}_t) & k_{\boldsymbol{x}_t}(\boldsymbol{x}_t, \boldsymbol{x}_{t-1}) \\ \mathbf{0}_{n\times K} & \mathbf{0}_{n\times K} & k_{\boldsymbol{x}_{t-1}}(\boldsymbol{x}_t, \boldsymbol{x}_{t-1}) \end{bmatrix}.$$

Then:

$$\mathbf{H}^i(\boldsymbol{x}_t) = \left[\frac{\partial h_1^i(\boldsymbol{x}_t)}{\partial \boldsymbol{x}_t}, \ldots, \frac{\partial h_K^i(\boldsymbol{x}_t)}{\partial \boldsymbol{x}_t}\right], \quad i = US, UK,$$

where, since $g'(z) = N(z)$, we get:

$$\frac{\partial h_\tau^i(\boldsymbol{x}_t)}{\partial \boldsymbol{x}_t} = N\left(\frac{a_\tau^i + \boldsymbol{b}_\tau^{i\prime}\boldsymbol{x}_t - \underline{r}^i}{\sigma_\tau^i}\right)\boldsymbol{b}_\tau^i, \quad i = UK, US. \tag{19}$$

Moreover:

$$\begin{aligned} k_{\boldsymbol{x}_t}(\boldsymbol{x}_t, \boldsymbol{x}_{t-1}) &= \frac{\partial k(\boldsymbol{x}_t, \boldsymbol{x}_{t-1})}{\partial \boldsymbol{x}_t} \\ &= \frac{\partial \boldsymbol{z}_t^{US\prime}}{\partial \boldsymbol{x}_t}\left(\boldsymbol{\Gamma}^{US\prime}\right)^{-1}\boldsymbol{\lambda}_{t-1}^{US} - \frac{\partial \boldsymbol{z}_t^{UK\prime}}{\partial \boldsymbol{x}_t}\left(\boldsymbol{\Gamma}^{UK\prime}\right)^{-1}\boldsymbol{\lambda}_{t-1}^{UK}, \end{aligned}$$

and

$$\begin{aligned} k_{\boldsymbol{x}_{t-1}}(\boldsymbol{x}_t, \boldsymbol{x}_{t-1}) &= \frac{\partial k(\boldsymbol{x}_t, \boldsymbol{x}_{t-1})}{\partial \boldsymbol{x}_{t-1}} \\ &= \mathbb{I}_{\mathbb{R}_+}(\delta_0^{US} + \boldsymbol{\delta}_1^{US\prime}\boldsymbol{x}_{t-1} - \underline{r}^{US})\boldsymbol{\delta}_1^{US} - \mathbb{I}_{\mathbb{R}_+}(\delta_0^{UK} + \boldsymbol{\delta}_1^{UK\prime}\boldsymbol{x}_{t-1} - \underline{r}^{UK})\boldsymbol{\delta}_1^{UK} + \\ &\quad + \frac{\partial \boldsymbol{z}_{t-1}^{US\prime}}{\partial \boldsymbol{x}_{t-1}}\boldsymbol{\Lambda}^{US\prime}(\boldsymbol{\lambda}_{t-1}^{US} + \varepsilon_{t-1}^{US}) - \frac{\partial \boldsymbol{z}_{t-1}^{UK\prime}}{\partial \boldsymbol{x}_{t-1}}\boldsymbol{\Lambda}^{UK\prime}(\boldsymbol{\lambda}_{t-1}^{UK} + \varepsilon_{t-1}^{UK}) \\ &\quad - \frac{\partial \boldsymbol{z}_{t-1}^{US\prime}}{\partial \boldsymbol{x}_{t-1}}\boldsymbol{\Phi}^{US\prime}\boldsymbol{\lambda}_{t-1}^{US} + \frac{\partial \boldsymbol{z}_{t-1}^{UK\prime}}{\partial \boldsymbol{x}_{t-1}}\boldsymbol{\Phi}^{UK\prime}\boldsymbol{\lambda}_{t-1}^{UK} \end{aligned}$$

where $\mathbb{I}_{\mathbb{R}_+}(z) = 1$ if $z > 0$, and 0 otherwise. Notice that

$$\frac{\partial \boldsymbol{z}^{US\prime}}{\partial \boldsymbol{x}} = \begin{pmatrix} \mathbf{I}_{n_0} & \mathbf{0}_{n_0 \times n_{US}} \\ \mathbf{0}_{n_{US} \times n_0} & \mathbf{I}_{n_{US}} \\ \mathbf{0}_{n_{UK} \times n_0} & \mathbf{0}_{n_{UK} \times n_{US}} \end{pmatrix} \quad \text{and} \quad \frac{\partial \boldsymbol{z}^{UK\prime}}{\partial \boldsymbol{x}} = \begin{pmatrix} \mathbf{I}_{n_0} & \mathbf{0}_{n_0 \times n_{UK}} \\ \mathbf{0}_{n_{US} \times n_0} & \mathbf{0}_{n_{US} \times n_{UK}} \\ \mathbf{0}_{n_{UK} \times n_0} & \mathbf{I}_{n_{UK}} \end{pmatrix}$$

are matrices that allow to select $\boldsymbol{z}^{US}$ and $\boldsymbol{z}^{UK}$ from $\boldsymbol{x}$.

- Given their predicted values, the updated values of the state variables are computed as:

$$\widehat{\boldsymbol{\xi}}_{t|t} = \widehat{\boldsymbol{\xi}}_{t|t-1} + \mathbf{P}_{t|t-1}\widehat{\mathbf{M}}_{t|t-1}(\widehat{\mathbf{M}}'_{t|t-1}\mathbf{P}_{t|t-1}\widehat{\mathbf{M}}_{t|t-1} + \mathbf{\Omega})^{-1}(\boldsymbol{y}_t - \widehat{\boldsymbol{y}}_{t|t-1}), \quad t = 1, \ldots, T$$

and

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\widehat{\mathbf{M}}_{t|t-1}(\widehat{\mathbf{M}}'_{t|t-1}\mathbf{P}_{t|t-1}\widehat{\mathbf{M}}_{t|t-1} + \mathbf{\Omega})^{-1}\widehat{\mathbf{M}}'_{t|t-1}\mathbf{P}_{t|t-1}, \quad t = 1, \ldots, T$$

- Given their updated values, the predicted values of the state variables are computed as:

$$\widehat{\boldsymbol{\xi}}_{t+1|t} = \boldsymbol{\mu}_\xi + \mathbf{\Phi}_\xi \widehat{\boldsymbol{\xi}}_{t|t}, \quad t = 1, \ldots, T-1$$

and

$$\mathbf{P}_{t+1|t} = \mathbf{\Phi}_\xi \mathbf{P}_{t|t} \mathbf{\Phi}'_\xi + \mathbf{\Sigma}_\xi, \quad t = 1, \ldots, T-1$$

- Finally, the likelihood function is computed using the recursive factorization:

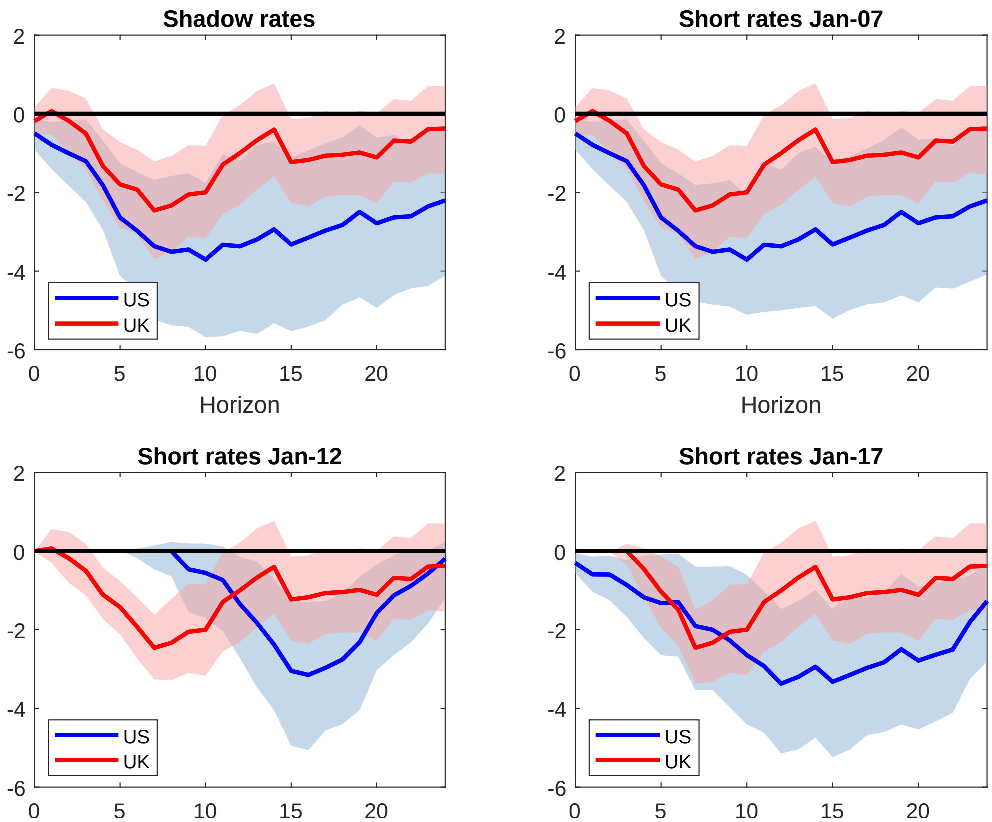
$$\boldsymbol{y}_t | \mathbf{Y}_{t-1} \sim \mathcal{N}(\widehat{\boldsymbol{y}}_{t|t-1}, \widehat{\mathbf{M}}'_{t|t-1}\mathbf{P}_{t|t-1}\widehat{\mathbf{M}}_{t|t-1} + \mathbf{\Omega}), \quad t = 2, \ldots, T.$$

B Model with two global factors, one US factor and one UK factor

In the main text we show that four factors are needed to explain the two yield curves (Table 1). Henceforth, the results presented in the paper rely on the baseline 4-factor model specification with 3 global (US) and 1 local (UK) factor. One obvious alternative is to allow for two global factors and one US-specific local factor, so-called SRTSM-211. Note that, in this case, both the UK and US yield curves are driven by three factors, while in the baseline SRTSM-301 specification, the UK curve was explained by four factors.

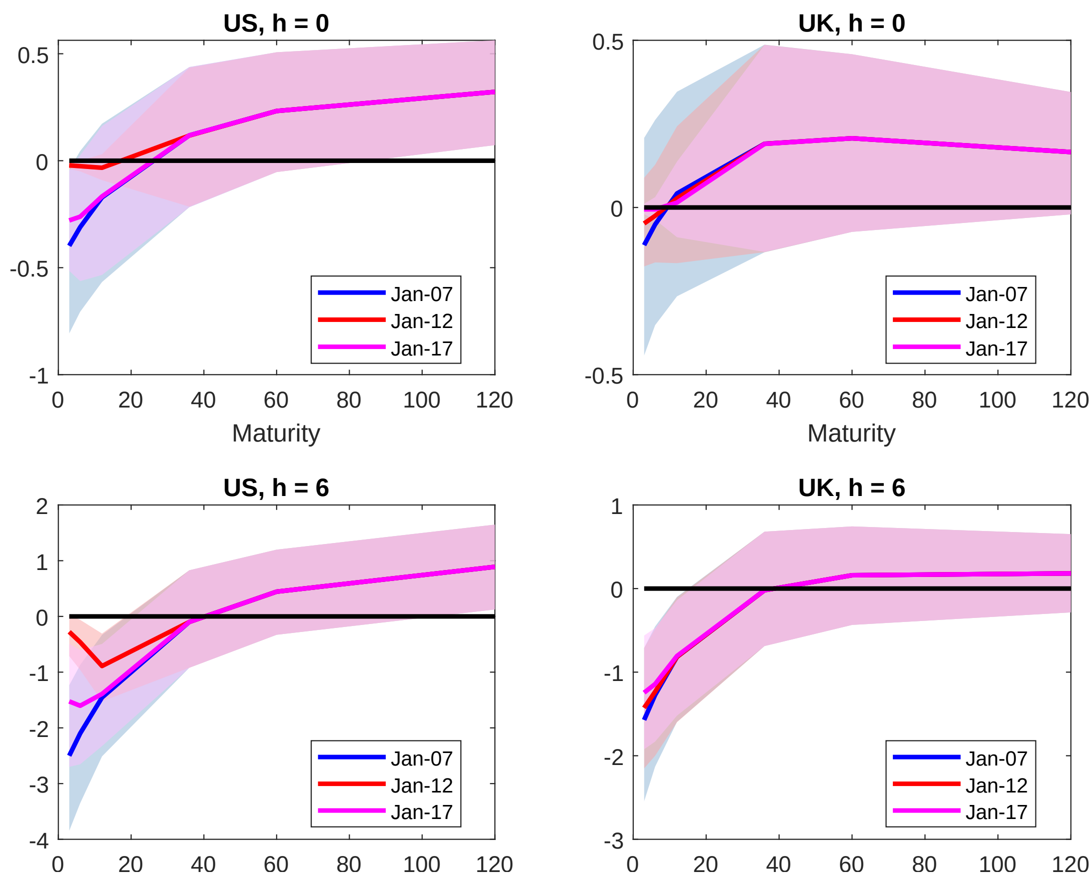
Estimating SRTSM-211 model, in which movements in individual curves were driven by three factors provides a similar fit to that of the baseline specification. The implied responses to the target and path US monetary policy shocks are also robust to the change in the factor structure: Figures 8, 9 and 10 show similar patterns to correspondingly those depicted by Figures s 4, 5 and 7 in the main text.

Figure 8: Model 211: shadow short rate and short rate responses to -50bp US target shock



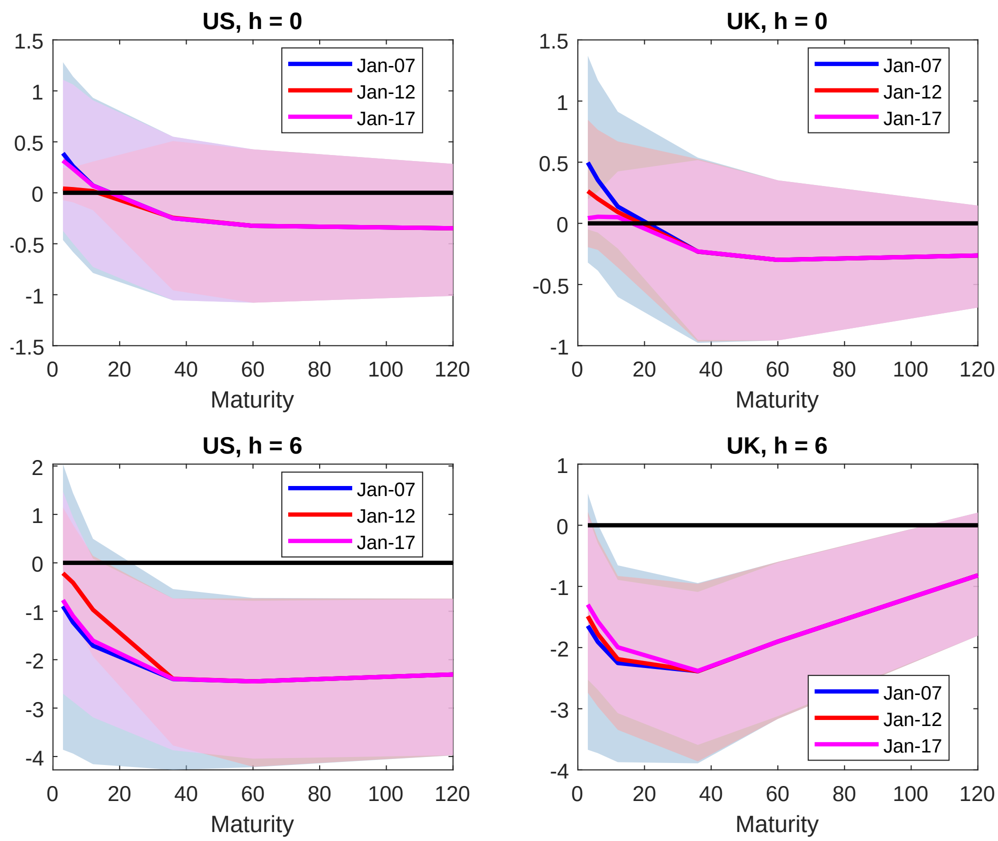
Impulse responses of the shadow short rate (top left plot) and the short rates on three dates (Jan 2007, Jan 2012 and Jan 2017) to a -50bp target shock of Kaminska et al. (2021) implied by the SRM with 2 global, 1 US and 1 UK factors. The shaded areas indicate the 80% confidence intervals.

Figure 9: Model 211: forward curve response to -50bp US target shock



Impulse responses of forward rates on three dates (Jan 2007, Jan 2012 and Jan 2017) to a -50bp target shock of Kaminska et al. (2021) implied by the model with 2 global, 1 US and 1 UK factors. The shaded area indicates the 80% confidence interval.

Figure 10: Model 211: forward curve response to -25bp US path shock



Impulse responses of forward rates on three dates (Jan 2007, Jan 2012 and Jan 2017) to a -25bp path shock of Kaminska et al. (2021). The shock is normalised to have an impact response of the US 36-month forward rate of -25bp in normal times. The shaded areas indicate the 80% confidence intervals.

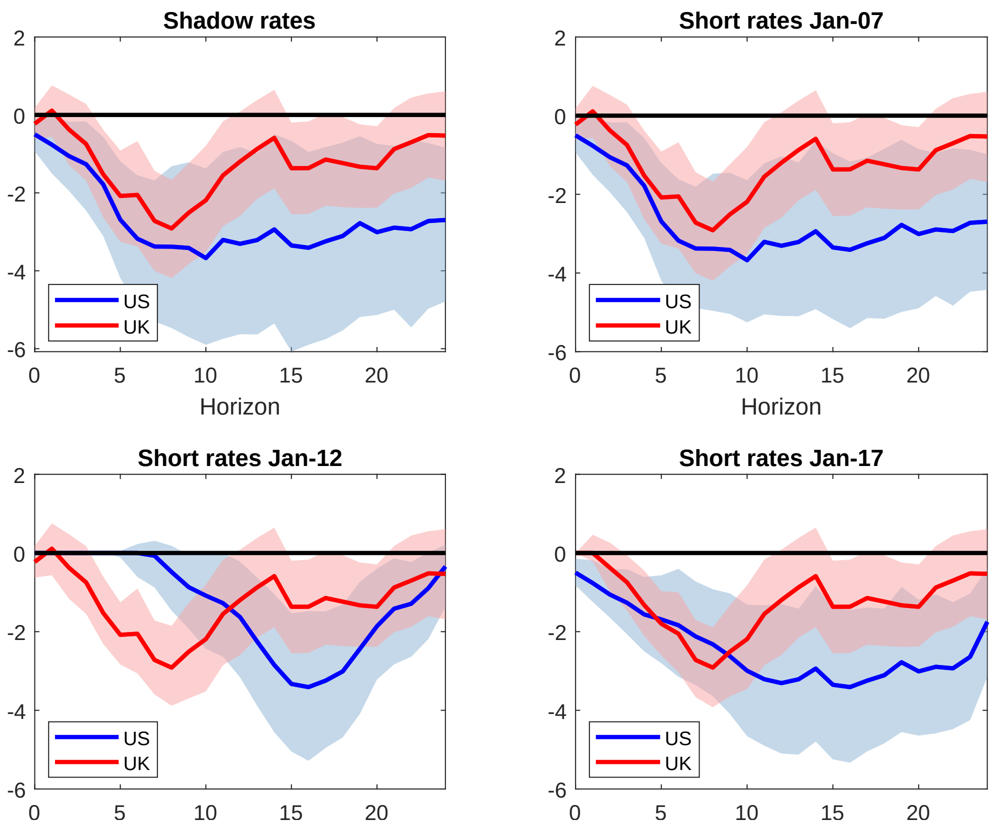
C Responses from the SRM without FX

This Appendix analyses transmission of monetary policy shocks via an alternative joint shadow rate term structure model specification, where we do not include FX depreciation rate in the model. Although, as shown in the main text, including FX data in the measurement equation provides a slightly lower RMSEs for the interest rates fit (Table 5) and is also better in capturing long rate correlations (Table 6), the recent literature finds the FX bond disconnect (see Chernov and Creal 2023), suggesting the inability of bonds to span exchange rates. Thus, excluding exchange rate information presents a natural robustness check for our model. In what follows, we repeat the response analysis of Section 6.2. in this alternative setup.

First, Figure 11 reports the responses of UK and US shadow and short rates to -50bp US target shock. The comparison with Figure 4 in the main text shows that the results are very similar for both model specifications, suggesting that the effect of non-linearity on short and shadow rates introduced by the ELB is a robust result. Similarly, Figures 12 and 13 are counterparts of Figures 5 and 7, respectively. They also confirm the robustness of our results.

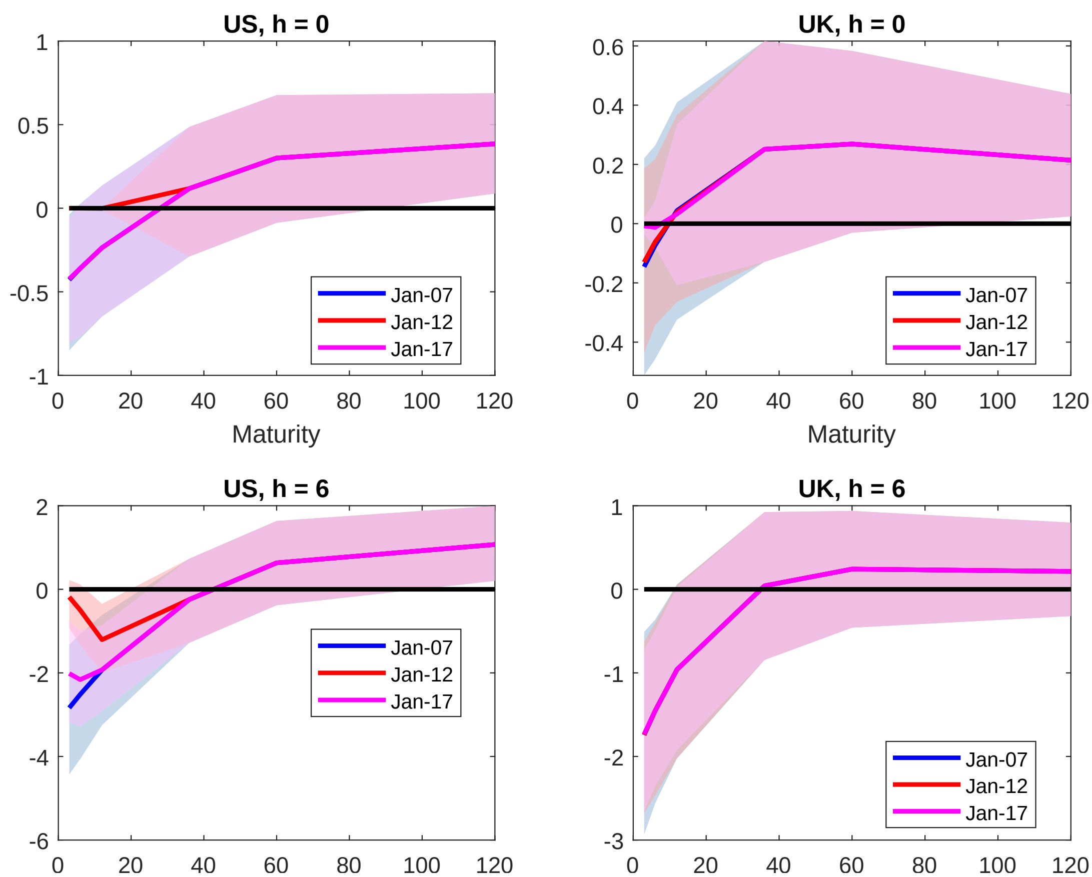
In sum, the asymmetric and non-linear effects of policy shock transmission introduced by the lower bound is a robust result, which is not affected by including or excluding FX information into or from the model.

Figure 11: SRM without FX: shadow short rate and short rate responses to -50bp US target shock



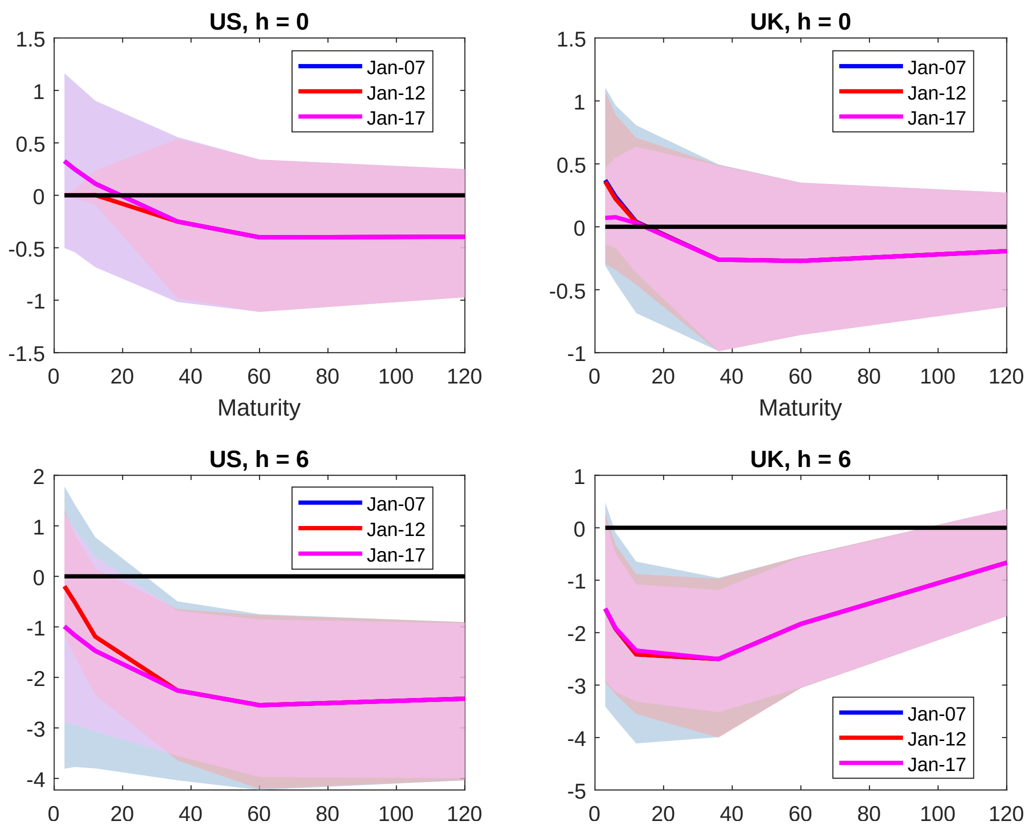
Impulse responses of the shadow short rate (top left plot) and the short rates on three dates (Jan 2007, Jan 2012 and Jan 2017) to a -50bp target shock of Kaminska et al. (2021) implied by the SRM without FX. The shaded areas indicate the 80% confidence intervals.

Figure 12: SRM without FX: forward curve response to -50bp US target shock



Impulse responses of forward rates on three dates (Jan 2007, Jan 2012 and Jan 2017) to a -50bp target shock of Kaminska et al. (2021) implied by the SRM without FX. The shaded areas indicate the 80% confidence intervals.

Figure 13: SRM without FX: forward curve response to -25bp US path shock



Impulse responses of forward rates on three dates (Jan 2007, Jan 2012 and Jan 2017) to a -25bp path shock of Kaminska et al. (2021). The shock is normalised to have an impact response of the US 36-month forward rate of -25bp in normal times. The shaded areas indicate the 80% confidence intervals.